EXPERIMENTS IN COMPUTER AIDED DESIGN

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DEPARTMENT OF MECHANICAL ENGINEERING

SUBJECT : COMPUTER AIDED DESIGN

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INDEX

EXPERIMENT NO. - 01

AIM: TO STUDY ABOUT CAD SYSTEM

- **TECHNOLOGY OF CAD**

CAD technology makes use of the computer to create drawings of parts and assemblies on computer files which can be further analyzed and optimized. The functional, ergonomic and aesthetic features of the product can be evaluated on the computers. This has been made possible through the use of the design workstations or CAD terminals, and graphics and analytic software, which help the designer to interactively model and analyze objects or components. CAD can be put to a variety of uses, some of which are listed below:

(i) Create conceptual product model/models.
(ii) Editing or refining the model to improve aesthetics, ergonomics and performance,
(iii) Display the product in several colors to select color combinations most appealing to customers,
(iv) Rotate and view the objects from various sides and directions,
(v) Create and display all inner details of the assembly.
(vi) Check for interference or clearance between mating parts in static and/or in dynamic situations,
(vii) Observe functional aspects of relative movements of various elements or assemblies,
(viii) Analyze stress, static deflection and dynamic behavior for different mechanical and thermal loading configurations and carry out quickly any necessary design modifications to rectify deficiencies in the design.
(ix) Study the product from various aspects such as material requirements, cost, value engineering,' manufacturing processes, standardization, simplification, variety reduction, service life, lubrication, servicing and maintenance aspects etc.
(x) Prepare detailed component drawings giving full details of dimensions, tolerances, surface finish requirements, functional specifications etc.
(xi) Prepare Assembly drawings depicting the orientation of components, assembly procedures and requirements, and incorporation, as required, such details as hydraulic or electrical connections.
(xii) Prepare exploded views of the assemblies for service and maintenance manuals. These views could be so oriented as to provide better visibility and improved comprehension of the design,
(xiii) Plot or print the picture/drawing stored in a computer file or the computer screen on different media,
(xiv) Store the database of the object, part or drawing in a magnetic disc or tape for retrieval at a later date for use in some other design or for modification of existing design.

The above description reveals that CAD techniques give the design engineer a powerful tool for graphical and analytical tasks. Modern CAD systems are based on interactive computer graphics, in which the computer is employed to create, transform and display geometric data. The designer communicates data and commands to the computer through the several input devices, to create an image or model on the computer screen by entering commands to call and activate the required software subroutines stored in the computer. In a 2-dimensional drafting system the images are constructed out of basic geometric elements or entities like points, lines, arcs, circles etc. These images can then be modified, rotated, scaled or transformed in several ways depending upon the designer's requirements.

- **REASONS FOR IMPLEMENTING CAD**

There are several reasons for implementing a computer aided design and drafting system in a design office.

1) To Increase the Productivity of the Designer

CAD helps the designer to quickly visualize the product and its component subassemblies and parts. This reduces the .time required to synthesize, analyze and document the design. The result is not only lower design costs but also shorter product development times.

2) To Improve the Quality of Design

A CAD system permits a thorough engineering analysis within a short time using software and thus a large number of design alternatives can be investigated. Design errors are reduced by the in-built accuracy of calculations and checks available in the system. These factors lead to improvement in the quality and accuracy of design.

3) To Improve Documentation

The use of CAD system provides better engineering drawings, more standardization in the drawings, better documentation of the design, fewer drawing errors and greater legibility for the drawing.

4) To Create a Database for Manufacturing

In the process of creating the database for the product design (geometry and dimension of components, bill of materials etc.) much of the required database to manufacture is also created which can be accessed for several Computer Integrated Manufacturing (CIM) applications like CNC programming, programming of robots, process planning etc.

- **HISTORICAL DEVELOPMENT**

The evolution of CAD has been largely dependent upon the developments in computer graphics. The early work in this direction was done at MIT, USA in connection with the development of computer graphics under a US air force sponsored project. Another early use of computer graphics as a human interface was "SAGE" system used in conjunction with radar communication during mid 50's. MIT developed the TX1 computer in early 60's and the first DEC (Digital Equipment Corporation) interactive graphics system was modeled on TX1. DEC introduced the first intelligent graphics terminal in 1968 (DEC 338).

Dr. Ivan Sutherland developed the "Sketch pad" at MIT in 1961. At the same time late Mr. Steve Coons started the development on surface patch technique. CAD technique subsequently was used by General Motors for the design of cars. IBM introduced the first graphics terminal in 1964. Several aircraft companies entered the field of computer graphics during 60's. Notable among them are Lockheed, McDonnel Douglas and Boeing. IBM organized a program called Project Demand involving Lockheed, North American Rockwell, McDonnel Douglas, Rolls Royce and other companies in an effort to evolve CAD and ultimately CAM. Many other computer manufacturers like Control Data Corporation, Univac, Ferranti, DEC, Tektronix, Adage, Applicon, Calma, Lundy, Megatek, Ramtek, Data General, Sum-magraphics, Sun, Apollo, Intergraph; Hewlett-Packard etc. entered the CAD field.

Today, computer graphics is a fast developing technology with a turnover over several billion rupees with a large number of manufacturers.

- **THE DESIGN PROCESS**

Before examining in detail the several facets of computer aided design, let us first consider the manual approach to design process, which is illustrated in Fig. 1.4. This process is basically a sequential process.

The demand for a product is recognized or anticipated by marketing department or R & D. Next step in the design process involves drawing up thorough specifications of the item to be designed. This specification involves physical and functional characteristics, cost, quality and performance.

Conceptual and detailed designs form very critical aspects of the design process in which the creativity and ingenuity of the designer play a major role. These phases of the design process are highly iterative. At the conceptual design stage, designers try several alternative forms and shapes for the product. Several variations of the product are studied to arrive at a compact and economical functional design. In automotive and aircraft industries, this process takes years. The initial design is often modified several times based on the data from analysis and optimization. This process of design development is called Computer Aided Engineering (CAE).

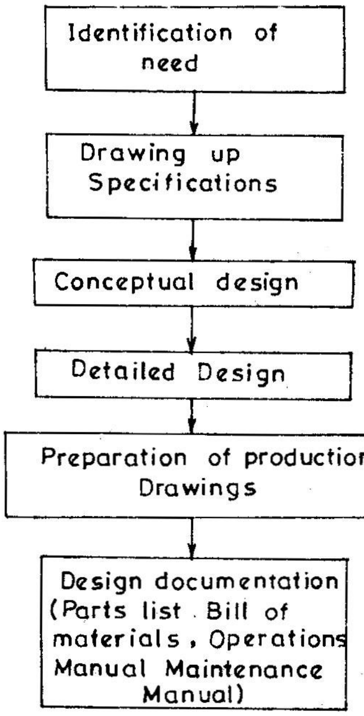


Fig. A - Conventional Sequential Design Process

The conceptual design packages, solid modeling packages and packages like finite element analysis and kinematical analysis available today can accelerate the design process at this stage. The CAE process enables the design and development engineer to perfect the design to a great extent. Thus the initial prototype is replaced by a mathematical model in the computer which can be tested for strains, deflections, vibration response etc on the graphics workstation itself. This phase also yields vital information which may help the designer to make suitable alterations to improve the product. The CAE technique enables comparative evaluation of prototypes at the graphics workstation. The components, subsystems and systems so synthesized are then evaluated, often by fabricating and testing a prototype model to assess the product's operating performance, quality, reliability and other relevant criteria. The final phase of design includes preparation of working-drawings, material specifications, bill of materials, part lists, process planning and cost estimation. In engineering design, the approach has been to synthesize a preliminary design and then subject that design to suitable forms of analysis, which may involve very elaborate calculations. Often subjective judgment plays an important role in shaping a design. One can, however, note the iterative nature of the design process and the amount of time and labor involved in going through this process to evolve an optimum design (Fig. A).

The design environment today can be personal computer or a customized workstation. The leaders in the workstation market today are SUN Microsystems, Hewlett-Packard, IBM, DEC, Intergraph, and Silicon graphics.

It is interesting to review the contribution of CAD to microelectronics industry. In the mid-60's US government sponsored projects for the development of the first software tools for integrated design to support custom 1C need. Early software tools developed were for logic block simulation and placement, routing and folding. Several software tools for modeling and analyzing transistor circuit performance were developed by late 60's. Software tools were also evolved for logic design, design rule checking, etc. Tools for simulation, analysis, cell placement and interconnect routing were available by 1982.

- **APPLICATION OF COMPUTERS TO DESIGN**

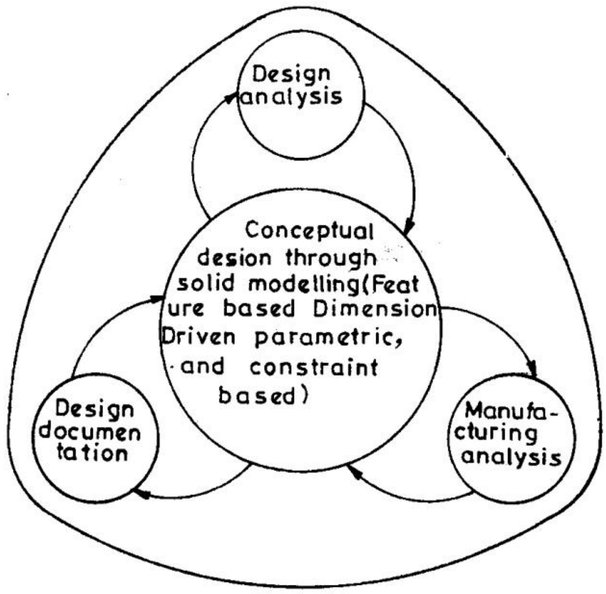


Fig. B - Concurrent (simultaneous) approach to design using CAD/CAM.

Computers are increasingly used in mechanical engineering, electronics engineering, architecture and other engineering disciplines. The following section describes the approach to design using CAD in the mechanical and electronic industry.
Various design related tasks which are performed by a modern mechanical engineering CAD system are shown in Fig.B It can be seen that the involvement of the design engineers is the same as shown in Fig. but at the various stages of design, the computer can be made use of to assist design process in a number of ways.

1) Modeling of the Design

Modeling involves the development of a mathematical description of the geometry of an object. The mathematical description allows the image of the object to be displayed and manipulated on a graphics terminal. The modeling can be in 2-D, or 3-D. In three dimensional modeling one has the option of 3-D wire-frame modeling, surface modeling or solid modeling. As a matter of fact with the use of solid modeling software, a design can be directly conceived at the graphic terminal itself, using feature based approach.

2) Engineering Design and Analysis

The engineering analysis may involve calculations of stress and deflections, heat transfer calculations or computation of natural frequencies, mode shapes, acceleration, velocity and displacement response or other parameters. Analysis can be carried out using standard CAD software like finite element modeling and analysis or by software developed internally in the design offices. Most CAD software suites incorporate engineering analysis software to carry out finite element analysis and to determine mass properties of the component. The extensive use of post processors enables visualization of the product under stress and strain. For the design of mechanisms, kinematic analysis software can be used. Several application specific software packages are also available now.

- ## BENEFITS OF COMPUTER AIDED DESIGN

The benefits of computer aided design are both tangible and intangible. The intangible benefits are reflected in improved work quality and more pertinent and usable information. Some of tangible benefits are discussed below:

1) Productivity Improvement in Design

CAD helps in increased design productivity by reducing the time for developing conceptual design, analysis and drafting. It is also possible to reduce the manpower requirements for a given project. The improvement in design productivity depends on:
Complexity of the drawing
Degree of repetitiveness of features in the designed parts
Degree of symmetry in the parts
Extensive use of library of user defined shapes and commonly used entities.

2) Shorter Lead Times

Interactive CAD is inherently faster than traditional manual design process. CAE tools reduce the number of iterations. It speeds up the task of preparing bill of materials. Using CAD systems a finished set of drawings and documentation can be prepared in a relatively short time. Shorter lead times in design result in reduction of the elapsed time between customer order and delivery of finished product. In preparing quotations, CAD enables faster response to customer enquiries by speeding up proposal stage designs.

3) Flexibility in Design

Apart from generating designs with improved accuracy, CAD offers the advantage of easy modification of design to accommodate customer's specific requirements.

4) Design Analysis

The design analysis routines available in a CAD system help to optimize the design. The use of analysis software (finite elements analysis and kinematic analysis) reduces the time and improves design accuracy and reduces the material used. Calculation of mass properties can be made almost instantaneously.

5) Fewer Design Errors

Interactive CAD systems have built-in capability for avoiding errors in design, drafting and documentation. These errors occur quite naturally during manual operation.

Errors are further avoided because interactive CAD systems perform time consuming and repetitive functions such as multiple symbol placement.

6) Standardization of Design, Drafting and Documentation

The single database and operating system used in CAD provide a common basis for design, analysis and drafting process. With interactive computer aided design drawings are standardized as they are drawn. It is also possible to reuse previous modules in developing a range of products.

7) Drawings are more understandable

With the increase in the use of 3D views and solid modeling, it has become easier to comprehend the features of the component readily. One does not have to reconstruct mentally the solid shape from 2D objects. Many software packages allow 3D view generation from a 2D model. This has several advantages from the manufacturing point of view.

8) Improved Procedures of Engineering Changes

Control and implementation of engineering changes can be significantly and improved with computer aided design. Original drawings and reports are available and easily accessible in the design office. Revision information can be retained and new drawings with changes can be created without destroying previous features.

INPUT DEVICES

These are the devices through which the user/ operator communicates with the computer for feeding it with the necessary information, both graphical and alphanumerical as required. The various devices used are:

- Keyboard,
- Mouse,
- Light pen,
- Joystick,
- Digitizer, and
- Tablet.

The keyboard is the most basic input medium for all computers. The layout of keys on a keyboard generally consists of the traditional typewriter keys together with some special keys, which are used for controlling the execution of the program or the screen display (cursor movement). The presence of a higher number of keys would facilitate the interaction. But for CAD/CAM applications, a keyboard itself may not be sufficient by virtue of its being a digital interface. Hence, in addition to the keyboard, other analogue input devices are also used for CAD/CAM applications. These devices may be used for entering graphic data in a convenient form or for selecting an item from the menu displayed on the screen.

The mouse is a pointing device, which has been gaining importance with the advent of the microprocessors, and the pull-down menus associated with the application software. The mouse operates on three basic principles—mechanical, optical and opto-mechanical. The mechanical mouse contains a free floating ball with rubber coating on the underside (**Fig. 1**) which, when moved on a firm plane surface, would be able to follow the movement of the hand. The motion of the ball is resolved into X- and Y-motions by means of the two rollers pressed against the ball. They in turn control the cursor on the screen, which can then be utilized for any desired applications by means of the clicking of the buttons on the mouse. This can only suffice to point, on the screen but not for giving positional data. Further, the mouse is a relative device and not an absolute pointing device as the digitizers to be discussed later. Many a mouse available in the market contains two buttons for its operation, though, mice with a larger number of buttons (3 and 4) are also available but have gained only limited acceptance.

In the case of the optical mouse, a special reflective plane surface with etched fine grids is required. The LEDs present inside the mouse (in place of the rubber ball) would reflect the number of grid lines crossed in the X and Y directions, thereby showing the distance moved. The life of the optical mouse is high since it has no moving parts, but it has not gained as much acceptance as the mechanical mouse because of the special surface needed for its operation. The operation of the opto-mechanical mouse is similar to that of the mechanical mouse, but the position resolvers used are based on the optical principle.

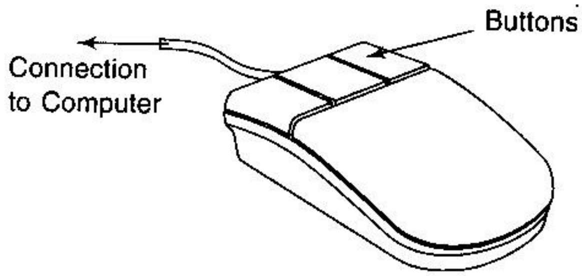


Fig. 1 - Mouse

A light pen resembles a fountain pen in the method of holding, but it works on the principle of light rather than ink, hence the name. Light pens are not used for writing on the screen as is erroneously believed by many, but actually only to detect the presence of light on the screen, as shown in **Fig. 2**, with the help of a light-detecting resistor. Their normal use in graphic applications is to identify objects or locations on the display screen for possible graphics handling. These are to be used only with refresh type display devices. The resolution of the light pen is poor, as the field of view of the photo-sensitive element is conical.

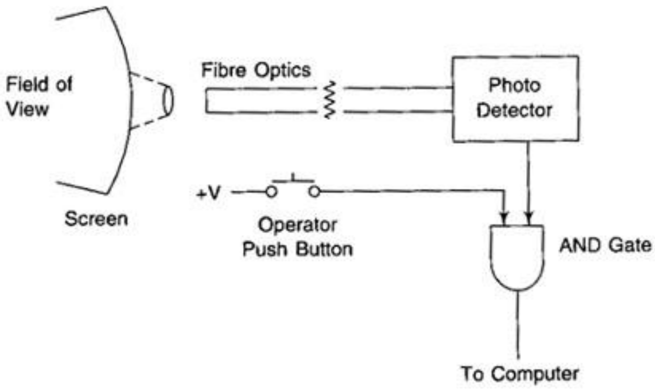


Fig. 2 - Light Pen

Since the light pen points to the graphic display directly, it is a natural graphic interactive tool. However, as the operator has to hold the light pen against the gravity along with its cable connecting the graphic adapter card for making any selection, ergonomically it is inconvenient to use it over long periods.

A joystick can also be used to control the on-screen cursor movement as a mouse does. A joystick can indicate the direction, speed and duration of the cursor motion, by the movement of the stick, which contains a ball seated in a spherical cavity, held in position by the operator. Generally, the response of the joystick would be quicker compared to other cursor-tracking devices. Hence, they are more suited for video games.

A digitizer is the most widely used input medium by the CAD designer. It is used for converting the physical locations into coordinate values so that accurate transfer of data can be achieved. Very high resolution in the order of 0.1 mm or better can be achieved. A tablet is essentially a low resolution digitizer having a small work area. The work area corresponds to the full CRT screen.

The designer can work with any pointing device similar to a pen, and do normal writing on the tablet as if he were doing so on a drawing board. The movement of the pen tip would be communicated onto the screen, which the designer can modify depending on the software at his disposal. Since it gives natural feel to a designer for free form sketching (which can be straightened if necessary by the software), it is generally preferred as a pointing device in CAD applications. Another kind of pointing device used in tablets is a puck, which has a cross hair line cursor and a number of buttons, as shown in **Fig. 3** as used normally with other digitizers. This would be useful for the menu selection. The buttons can be assigned for different auxiliary functions.
Glass Window Cross-hair

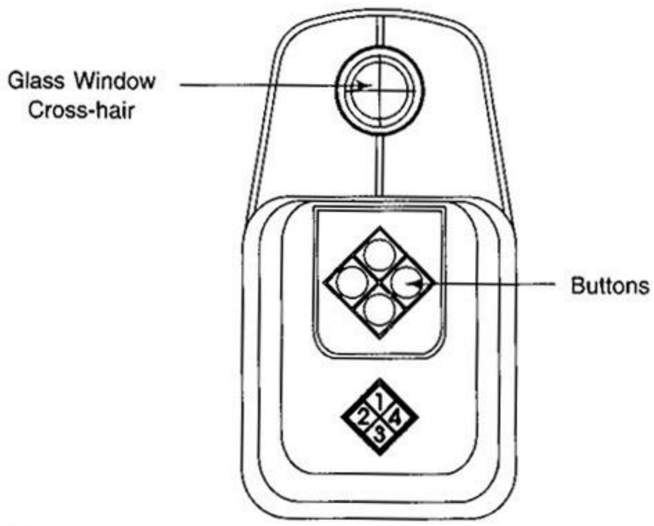


Fig. 3 Puck or Pointing Device used with Tablets and Digitizers

A digitizer consists of a rectangular smooth surface as a draughting board, as shown in **Fig. 4** Underneath this surface would be a position-sensing mechanism. The designer interacts through the hand-held locator (or puck as shown in **Fig. 3**) which contains a number of buttons. The designer can move the puck to the desired position and then by pressing one of the buttons to initiate a certain action. A digitizer is an absolute measuring device.

Fig. 4 Digitizer

Electromagnetic force is the measuring means that is most generally employed in digitizer construction. It contains a printed circuit board, which is etched with evenly spaced traces on both sides, one representing the X-axis and the other the Y-axis. The locating device used (such as puck or a pen type) would contain a coil which can act as a receiver or a transmitter. In the transmitter mode, electronic signals on the board can be measured for their strength, and the highest strength signifies the location of the desired point. The digitizer controller would measure the position of X- and Y-axes alternatively.

The other technology used in the digitizer is resistance. The digitizer is made of two sheets separated by a number of spacer dots. One sheet is evenly deposited with a resistive film, whereas the other is coated with a conductive film. A small voltage is applied across the resistive film. When pressure is applied by the puck or stylus on the digitizer pad, the potential of the conductive sheet would be proportional to the distance from the end, which gives the locational data. The measurement is again to be done alternatively in X- and Y-axes. The same resistive technology may be effectively used in touch pad type applications.

In a touch screen, a transparent digitizer composed of conductive glass film is present on top of the CRT screen, such that when pressure is applied at a given point on the screen, it can be sensed. This is comparable with the application of a light pen.

The digitizer comes in a large number of sizes from 250 x 250 mm to as high as 1000 x 3000 mm. The quality of a digitizer can be measured in terms of resolution, accuracy, linearity and repeatability. Linearity is the variability of accuracy over large areas of the digitizer. Digitizer communicates with the computer in single point mode, as the coordinates of the point are transmitted when a button on the puck is pressed. This is used for digitizing discrete points such as an already existing drawing. However, when continuous sets of points are required, the puck can be moved continuously over that line, and the coordinates of the points sent to the computer at a specified sampling rate, such as 200 coordinate points per second. Some typical digitizer specifications are presented in **Table 1**.

Table - 1 Digitizer Specifications

	GRAPHTEC	*Drawmaster*	*Summa Sketch III*
1. Digitizing method	Electro-magnetic	Electro-magnetic	Electro-magnetic
2. Digitizing area, mm × mm	380 × 260	1117 × 1524	305 × 305
3. Resolution, mm	0.1	0.025	0.01
4. Accuracy, mm	0.5	0.05	0.254
5. Reading speed positions/second	52	60	—
6. Interface	RS 232	RS 232	RS 232
7. Maximum transmission speed for serial communication, baud rate	9600	—	—
8. Operating modes available	Point, stream, trigger	—	Point, stream, trigger

Another class of input devices sometimes used is the 3D digitizer, which has the ability of converting any 3D object into its dimensional form. These are also some times referred to as space digitizers. The method of digitizing could be to manually move a stylus along the desired 3D object, but this is time-consuming. Therefore, optical scanning is done for such imaging work. A thin plane of light is projected onto the 3D image which, when reflected, is received by a camera through a system of mirrors and is then recorded.

DISPLAY DEVICES

The display device forms the most important element in a CAD/CAM system, since on this most of the design work and simulation of manufacturing can be graphically displayed. The display media that are used are:

- Cathode ray tube (CRT) display,
- Plasma panel display or
- Liquid crystal display (LCD).

Of these three methods, it is the CRT displays that are the most advanced and extensive in use, in spite of their bulkier size.

1) CRT Display

In a CRT display, the heated cathode emits electrons, which are formed into a stream, accelerated and focused onto a point on the display screen. The display screen contains a phosphor-coated surface as shown schematically in **Fig. 5**, which gets illuminated when the speeding electrons hit the surface, displaying the point. The electron beam is controlled by means of deflection plates for accessing any point on the surface of the screen. Changing the beam current changes the intensity of the spot created on the screen.

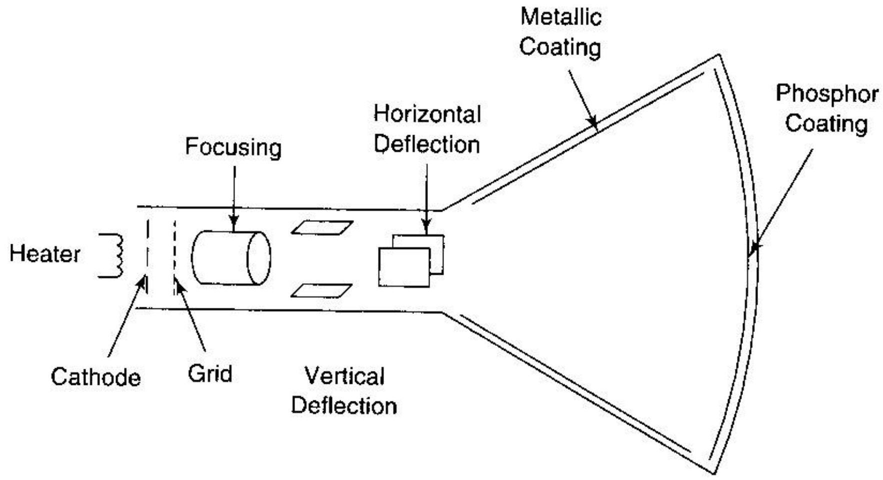


Fig. 5 The Cathode Ray Tube Principle

There are basically two types of image-drawing techniques that are used in graphic displays. They are:

• Stroke-writing and
• Raster scan

In a stroke-writing display, the electron gun directly draws the vectors on the screen to generate the image, whereas in the raster scan, the whole display surface is divided into a matrix of small dots called pixels (picture elements) and the electron beam scans the whole surface area line by line, as in that of a home television.

Irrespective of the writing technology employed, the phosphor glow created by the electron impingement on the picture screen is short lived. Therefore, some means are to be devised for overcoming this shortcoming and achieving a static image on screen.

One method of maintaining a static display is by using storage tube technology developed by The Tektronix in 1972 called the direct view storage tube (DVST). Here, the display is generated by the impingement of electrons as in the conventional CRT. However, a cathode grid would be part of the screen surface, which, once excited by the electron beam would continuously emit electrons which would maintain the image on the screen. This is desirable because there being no need for refreshing the image; substantial overheads on the display electronics are averted. The resolution that can be achieved is in the order of 3000 X 4000 addressable points on a 19 - inch tube and it results in a clear image without any flicker. This explains the popularity of this technology with the earlier CAD/CAM system.

However, disadvantage of this system is that once written, partial erasing of an image is not possible. Any necessary modifications could only be made after completely erasing the picture and then redrawing, which cost a lot of time, particularly for complex images if they were to be altered a umber of times. Animation is not possible, as it relies on the erasing and drawing of parts of the image on the screen. Also, the image can be obtained in monochrome only. Presently, this type of display device is almost obsolete as far as the CAD/CAM sphere is concerned, though some 'EKTRONIX terminals have achieved both color-filling as well as partial erasure facilities in the storage tubes.

The second method of technology used is that of direct stroke writing with a direct refreshing tube r vector refresh tube. In this, the image is generated on the screen by direct drawing of straight vectors on the screen. As the phosphor glow is short-lived, it is continuously refreshed by repeated stroke writing at a rate fast enough to eliminate flicker from the screen. To maintain a flicker free vision of the image, it is necessary to refresh the whole screen at a rate of about 60 times every second 50 Hz). Since the image is continuously refreshed, it is possible to erase and modify parts of the display to any extent desired. A major disadvantage of the vector refresh devices is that the display tarts to flicker if a large amount of data is on the screen. When the image contains more than 4000 vector inches of graphics, the refresh rate may drop below 30 Hz and the image would start flickering, these are also very expensive.

It is also possible to obtain color display in refresher tubes. Here, the phosphor coating on the screen contains three different dots (red, green and blue) arranged side by side at the same spot. The CRT contains three electron guns, each corresponding to the primary color. Information regarding the intensity of each of the colors would control its gun, the beams of all three guns being simultaneously focused onto the screen through a shadow mask from where they diverge to fall on the corresponding phosphor dots giving rise to the desired color and intensity on the screen. The glow of ach of the phosphors at a given point controls the color at that point. With only a red glow, a red dot will be displayed, but red and green in combination may present orange or yellow depending upon the strength of each color, which can be controlled by the beam strength. With proper control of the display electronics, it should be possible to display a large number of colors on the screen. Typically, 16, 64, *256* to 4096 or larger depending upon the capability of the display device.

In the raster scan displays, the complete screen is divided into a matrix of pixels from a typical 320 x 200) to as high as (1280 x 1024) or more as shown schematically in Fig. 6. Each square in the Fig. 6 represents one pixel. The electron beam generates a single dot at the centre of this square. Tie distance between the squares is called dot pitch and it indicates the fineness of the display. Typical dot pitches used is 0.28 mm in the current day low cost display monitors. However, a dot itch less than 0.25 mm is preferable for a sharper display image.

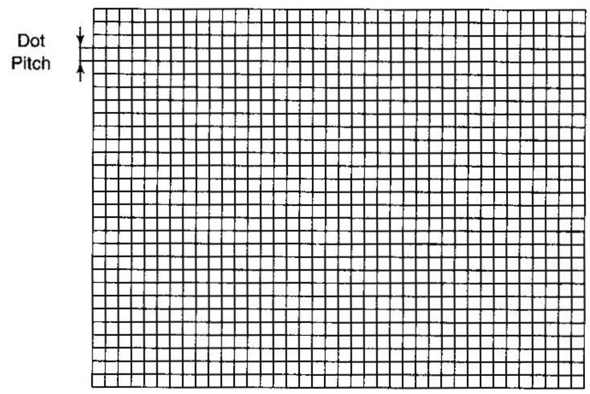


Fig. 6 Dividing the Screen into Small Points Called Pixel. Each Square Represents a Pixel

The display is generated by identifying which pixels need to be bright for a given vector and then le full screen display is obtained by scanning the complete screen horizontally line by line as shown in Fig. 7. This is similar to the process in the domestic television. The refresh rate is to be mainlined sufficiently high so that no flicker in the image is perceivable. This normally amounts to 60 times a second and is represented as the refresh rate 60 Hz. This means that the whole screen is to be completely written in l/60th of a second. This is called sequential or non-interlaced refreshing. The refresh rate also depends on the resolution of the screen, with higher resolutions requiring faster refresh rates. Typical refresh rates used in Pentium-based workstations is given Table 2**.**

The process of writing on a refresh type monitor requires that the electron beam will pass through all those points which will require being bright and as a result, the vectors in the drawings are to be converted into its equivalent pixel points. This process is termed as rasterisation. The raster images of lines and circles for typical orientations are shown in **Fig. 8** and **Fig. 9**.

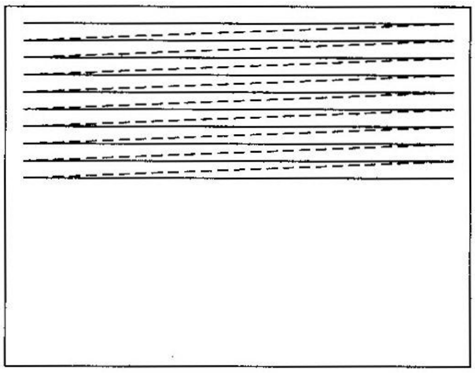


Fig. 7 Raster Scan Display for Continuous Scanning of Lines

In some low-cost display devices, decreasing the refresh rate to half at 30 Hz reduces the cost of the monitor. This gives rise to a flickering of the image as in each of the cycles only half of the screen image is refreshed, instead of the full one by omitting alternate lines. This is termed interlaced refreshing. In this mode, in the first cycle, alternate lines are refreshed as shown in Fig. 10 whereas in the second cycle the other lines are refreshed. This reduces the overheads or the display control and consequently the costs, but is not suitable for dynamic displays where the display changes fast.

Table 2 Typical Resolutions used in Pentium-based Workstations

Specification	*Resolution in pixels*	*Minimum Vertical refresh rate, Hz*	*Minimum Horizontal scan rate, kHz*
VGA	640 × 480	85	43
SVGA	800 × 600	85	54
XGA	1024 × 768	85	69
SXGA	1280 × 1024	75	80
UXGA	1600 × 1200	72	89

Fig. 8 Raster Display of Lines **Fig. 9 Raster Display of Circles**

A typical configuration of the raster scan display is shown in Fig. 11. Here, the frame buffer contains the complete dot by dot image of the display, which is required. From the frame buffer, this information is accessed by the sweeper, which in turn controls the display device. In the earlier systems, a part of the main computer stored the frame buffer, which imposed severe overheads on the performance of the system. As a result, the frame buffer is arranged separately along with a host processor to read from and write onto the frame buffer. The main processor controls the graphic processor in turn. If there were no separate graphic processor, then the main processor would directly communicate with the frame buffer.

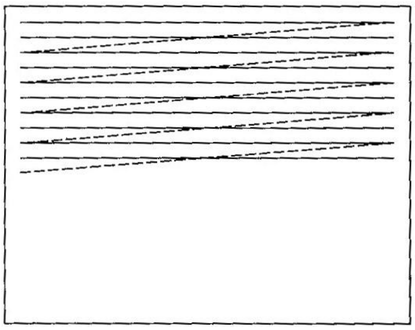


Fig. 10 Interlaced Scanning in Refresher Type CRT

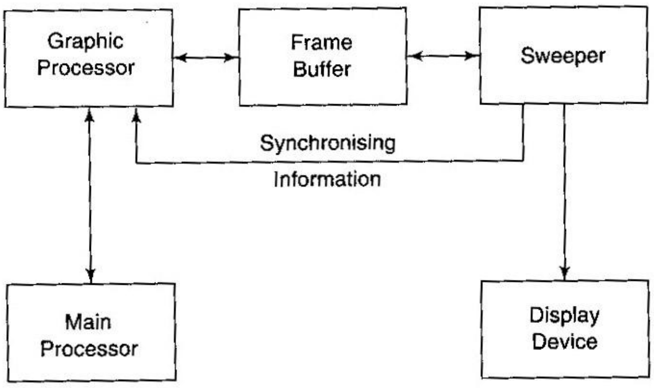


Fig. 11 Information Organization in a Raster Scan Display

What should be the capacity of the frame buffer? It depends upon the screen resolution required. Consider a monochrome screen resolution of 1024 x 1024. For each of the pixels to be represented on the screen one bit of information is needed, so the frame buffer capacity should be 1 M bits or 128 k bytes. However, for color display the required memory gets increased.

HARD COPY DEVICES

Once the output is finalized on the display device, it can be transformed into hard copy using:
• Graphical printers,
• Plotters, or
• Photographic devices.

1) Graphical printers

This is the fastest way of getting graphical output at low cost. The three principal technologies that are currently used are -

Impact Dot Matrix Printer

In this printer, the print head consists of a vertical bank of needles (9, 12 or 24) which moves horizontally over the paper. At each of the horizontal positions, any of the pins in the print head can make ink marks by hitting the paper through a ribbon. Thus, from the image on the screen, each of the pixels corresponds to the pin position as it moves over the entire page. The resolutions that are available vary but range from 60 dots per inch to 240 dots per inch. Their cost is comparatively low, but a major disadvantage is their noise because of the impact of the pins on the paper.

Thermal Transfer

This is similar to the dot matrix printer in operations, but is not to be confused with the normal thermal printers where sensitized paper is used for output. It uses a special ribbon positioned between the paper and print head. The ribbon is a roll of thin polymer material. Spots of the dye are transformed from the heat-sensitive ribbon to the paper underneath. Though they are relatively noiseless, with fewer moving parts and a low weight, the cost of the special ribbon to be used is high and it is still a developing technology. It is normally used for field applications where portability is required. Color thermal transfer printers from a number of vendors are to be found in the market with resolutions of 150 to 400 dots per inch.

Ink Jet Printer

This does not make use of any ribbon but shoots a jet of ink directly onto the paper, as the pin impact of the dot matrix print head. Normally, there would be a bank of ink nozzles positioned vertically, as the pins of an impact dot matrix print head. Otherwise, the rest of the mechanism is identical to the impact dot matrix printer. These are almost noiseless in operation. The print head of Hewlett-Packard ink jet printer consists of an ink cartridge holding 3 ml of ink in a cylindrical unit of 40 mm long which is enough to last for 500 A4 size papers. Ink from the reservoir terminates in 12 tiny holes, arranged vertically facing the paper. As the print head moves horizontally the droplets of ink are shot wherever required. Behind each hole is a small heating element which, when turned on vaporizes the ink partially, causing a force inside the cartridge, to eject the ink onto the paper. Resolutions can be in the order of 300 to 1200 dpi (dots/inch). The only requirement is that the paper used should be sufficiently absorbent, so that the droplet upon reaching the surface of the paper dries quickly. It is also possible to have full color printing using the ink jet technology by incorporating the primary color inks for each of the dots.

Each ink cartridge in a Lexmark 7000 series printer has an integral laser-crafted print head, which delivers 1200 x 1200 dpi. It consists of 208 nozzles in the print head of the black cartridge, and 192 nozzles in the color and photo cartridges. These nozzles let the 5700 create very small, precise dots of ink on the page, giving sharp text and fine line detail.

Canon bubble jet printers have a Black BJ Cartridge with 608 nozzles, the Color BJ Cartridge with 240 nozzles (80 nozzles x 3) and the Photo BJ cartridge with 480 nozzles (80 nozzles x 6) giving a print resolution of 1200 x 600 dpi.
The Calcomp Crystal Jet printer comes with four print heads, one for each of the four process colors: cyan, magenta, yellow and black with each print head having 256 nozzles. The heads are staggered in stair step fashion and mounted on a gold-plated nozzle plate. The print heads are spaced at l/180th-inch intervals, yielding a print swath of 1.4 inches per color. The printer can have a resolution of 180, 360 or 720 dpi.
One of the major problems with the ink jet printers is the cost of the ink jet cartridge per page used.

Laser Printer

The laser printer is essentially an electrostatic plain paper copier with the difference that the drum surface is written by a laser beam. A semiconductor laser beam scans the electro statically charged drum with a rotating 18-sided mirror (560 revolutions per minute). This writes on the drum a number of points (at 300 dots per inch) which are similar to pixels. When the beam strikes the drum in the wrong way round for printing a positive image, reversing it, then toner powder is released. The toner powder sticks to the charged positions of the drum, which is then transformed to a sheet of paper and bonded to it by heat. Though it is relatively expensive compared to the dot matrix printer, the quality of the output is extremely good and it works very fast at 8 to 16 pages per minute (A4 size). The current resolution available is 600 and 1200 dpi. Currently, the size is limited only to A3 or A4, though higher size electrostatic plotters with a slightly different technology are available but are very expensive. Though operationally the laser printers are fast, the copy cost is high, besides the size limitation.

Color Copiers

The first color copier was introduced by the 3M company in 1969. Since then, a number of companies such as Xerox, Canon and others have been periodically introducing their own models. These color copiers are essentially single color copiers but with the ability to change the developer housing or toner cartridge for using a different color. Thus, multiple passes of the same plot would enable one to obtain full color copies using these copiers.

2) Plotters

The plotter is the widely accepted output device for the final output. A large range of plotters of varying sizes and prices (see Table 3) are available. The accuracies achievable are very high and the plots can be made on all types of media such as paper, tracing paper and acetate film. Normally, all plotters will have a range of pens available, which can be changed under program control. Pens of any color or of different width writing can be used depending upon the output desired. The types of pens used are - fiber tip, roller ball or liquid ink. They are the slowest of all the high resolution plotters since the speed is dependent upon the pen's ability to draw lines.

Table 3 - Various Sizes for Plotters

Designation	*Size of drawing, mm*
A0	841 × 1189
A1	594 × 841
A2	420 × 594
A3	297 × 420
A4	210 × 297

Pen Plotters

Essentially, plotters are of two types - flat bed and drum type. In the flat bed plotter, the paper is held in a fixed position by means of vacuum or electrostatic force. The pen carriage would move in both X and Y axes for making the necessary plot. Its chief advantage is that any kind of paper is acceptable because of the simple nature of the plotter. However, these are very expensive compared to a similar size drum plotter. Moreover, the plot size is limited by the bed size of the plotter.

The drum plotter is slightly more complex. The Y motion of the plot being obtained by a rotating drum on which, the paper is held with the help of sprocket holes of a standard size. The X movement of the pen is arrived at by moving it in a direction perpendicular to the drum motion. As the paper is moved during the plotting, the size of the plot to be obtained can be varied by the program. As a result, the drum plotter would be cheaper for a given drawing size compared to a flat-bed plotter. The disadvantage lies in the use of special paper with proper sprocket holes for the plotter.

The plotters of the present generation are almost all of the drum type with the variation that the paper feeding is done by means of friction feed (pinch rollers). The pen plotters are low in initial purchase cost, and produce accurate drawings. However, they are slow and require a high level of maintenance.

Electrostatic Plotters

The electrostatic plotter uses the pixel as a drawing means, like the raster display device. The plotter head consists of a large number of tiny styluses (as high as 21,760) embedded into it. This head traverses over the width of the paper as it rolls past the head to make the drawing. These styluses cause electrostatic charges at the required dot positions to make the drawing. The resolutions available may be of 100 to 508 dots per inch. They are normally very fast with plotting speeds of 6 to 32 mm/s, depending on the plotter resolution. The speeds of different plotters vary with the acceleration of the pen as it draws the line and with the pen up/down cycle time. The factors to be considered while selecting the plotters are - plotting area, number of pens, type of pens used, drawing speed, resolution, accuracy and drawing protocol (plotting graphics language such as HPGL) it observed. Some typical plotter specifications are summarized in Table 4.

Table 4 Typical Plotter Specifications

	HP		*Graphtec*		*Bausch & Lomb*
	7475A	*7585B*	*MP3300*	*GP9001*	*DMP-42*
1. Paper size (Max)	A3	A0	A3	A0	A1
2. Pens	6	8	8	4	1
3. Resolution, mm	0.025	0.025	0.025	0.05	0.025
4. Repeatability, mm	0.1	0.1	0.1	—	0.127
5. Pen velocity, mm/s			400	350	116
pen down	381	600	—	—	—
pen up	508	600	—	—	—
6. Acceleration, max	2g	4g	—	—	—
7. Programming system	HP-GL	HP-GL	HP-GL	HP-GL	HP-GL
8. Method of paper feeding	Pinch roller	Pinch roller	Flat	Pinch roller	Pinch roller
9. Buffer capacity	1 k	18 k	24 k	—	—

3) Photographic Devices

The photographic recording devices are essentially cameras in front of a CRT display. The only difference is that they use a display device other than that used as the monitor. They normally have a smaller built-in screen inside the recorder, which is connected to the CPU through the serial communication port (RS232c). The image is obtained on a flat-faced monochrome CRT with a 4-position filter wheel to provide separate exposure for red, green and blue images. The image from the main computing system is received by these recorders, and then separate graphic processing is effected to remove the jagged edges on type fonts, circles and lines. It would normally be possible to enhance the image obtained on the display device through the software of the recording devices. The resolutions that are attainable vary from 500 lines to 4000 lines depending on the cost of the device and the presence of any other hardware that is required along with the device.

4) STORAGE DEVICES

Permanent storage of programs and of data generated during various sessions of CAD/CAM requires a large amount of storage space. This is normally denoted as auxiliary storage and the various devices used are:

- Floppy disks,
- Winchester disks,
- Magnetic tapes,
- Magnetic tape cartridges
- Compact disk ROMS, and
- DVD.

The floppy disk is the most convenient medium for handling data that is either temporary or permanent. It consists of a storage disk, which is magnetically coated on both sides. It is permanently enclosed in a square cover, lined internally with a special low-friction material. The floppy disk rotates at very high speeds, and the read/write head moves radially to read or write any data randomly from any location. The disk is divided into concentric rings called tracks. They are usually specified in tracks per inch (TPI) and are generally 48 TPI or 96 TPI. The tracks are further subdivided into radial sectors. Each sector can store information of about 250 bytes. They normally come in 3 standard sizes - 3.5, 5.25 and 8 inches. The storage capacities range from 360 k to 1.5 M bytes (formatted) storage. The formatted storage is the actual value of storage in bytes available for the user. By changing the recording technology, the floppy disk can store up to 10 M bytes per floppy. This technology is yet being developed and should be ready for use in the next few years.

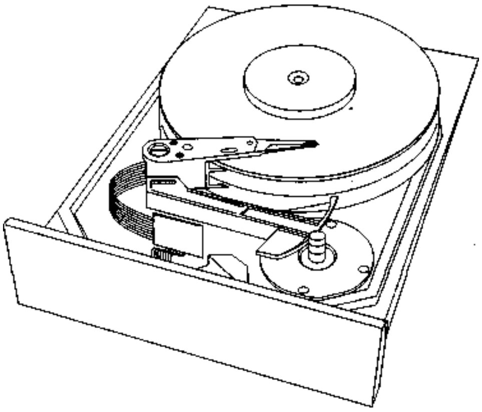


Fig. 12 Hard Disk

The Winchester disk is a thin, rigid metal disk on both sides of which the magnetic medium is coated. Several such hard disks are put together aligned on a central shaft disk pack as shown in **Fig. 12**. Separate read/write heads for each of the disks are permanently aligned and then the disk is sealed. Though there are more than one head, at any given time only one head would be accessing the disk for data input/output. This has a large storage capacity and has been extensively used because of its low access time, low cost and compact size. The disk is fixed inside the drive and therefore, the storage is fixed to the capacity by the drive. Typical storage sizes available presently are from 40 GB to 300 GB. However, removable disk packs are available whereby a large amount of data can be stored and used for back-up purposes.

The other kind of mass storage medium associated with computers is the 0.5 inch magnetic tape. The greatest limitation of magnetic tape is the serial nature of the storage, necessitating all the tape before to be wound for accessing any inside information. As a result, the magnetic tape is used only for data exchange or back-up, as a large amount of storage is possible. A typical storage density is 6250 bits per inch in 9 tracks. A 10.5 inch reel can store about 180 M bytes of data.

Another magnetic tape 0.25 inch in size in the form of a cartridge as in an audio cassette is also used by microcomputers and mini computers as a mass storage medium. These are essentially used as a back-up medium for taking periodical back-up from the hard disk drive.

Current developments in mass storage of very large capacities are based on the optical technology, rather than on the magnetic principle, which was used in all the previous storage devices. In this, a small aluminium compact disk 120 mm in size contains a number of pits in the size range of about 1.5 microns. A non-contact laser reads the information present on the disk. As a result, the storage densities can be very high. For example, a 120 mm compact disk can store 650 M bytes of data and is very easy to handle. Since the reading is done by the focused laser onto the pits, the information stored is permanent. For writing onto the disk, using the same technology, a high intensity laser is required to evaporate the material on the disk. This can only be done once and the drives used for this purpose are called WORM (Write Once and Read Many) drives and are now a common place in most of the computers.
The major disadvantage of this technology is that once the disk is written by making a pit, it cannot be erased. As a result, it becomes like a ROM and hence the name for these devices as CD ROM. Thus, these devices cannot be used as a regular auxiliary storage device with the computer but can be used for only data base purposes. However, current research provides a better medium for storage in the form of magneto-optical media. The recording medium orientation changes, causing the light passing through it to change in brightness. These media would provide a convenient means of erasing and storing large amount of data as a regular auxiliary storage medium.

The media of these types of storage systems consist of a polycarbonate platter in which is embedded a layer of aluminium backing, overlaid with a magneto-optical substrate. This substrate crystal orientation is changed magnetically to erase or write. The writing process consists of three passes. In the first pass to erase the written matter at a particular location, a magnetic field is applied on the platter for erasing the stated direction. A high power laser heats the platter to a temperature called Curie point at which the crystals in the substrate orient magnetically to the surrounding magnetic field, which is a 0. Thus, the data in the target location in the platter is erased. Next, the applied magnetic field orientation changes to the writing orientation, i.e. 1. The laser again heats, the same location to the Curie point such that the crystal orientation is now altered in the direction 1. The third pass is required to verify that what was written is the same as the data.

For the purpose of reading, the magnetic field is removed and a low intensity laser beam is directed, which passes through the substrate and is reflected by the aluminium backing. The crystal orientation in the magneto-optical substrate alters the polarization of the reflected beam, called the Kerr effect. The reflected beam is passed through a polarizing filter onto a photo detector the intensity of which determines whether a 0 or 1 is present at the target location. Presently, commercial drives are available which can store 650 MB on a 5.25 inch removable optical disk using this technology. This can also be read by modern CD-ROM drives. These are called CD-RW drives and are currently available. These are slowly replacing the existing floppy diskettes for data transfer and archiving.

Another storage medium which is becoming increasingly popular is the DVD originally named "Digital Video Disk," then "Digital Versatile Disk," and now simply "DVD". There is no official three-word equivalent to DVD. Like CD-ROM, the DVD is read by an infrared laser focused through a protective plastic layer onto the disc's reflective layer. The transparent layer is 1.2 mm thick on a CD-ROM, but only 0.6 mm on a DVD-ROM. The beam reflects off pits burned into the reflective layer by the recording laser and is passed through optics to the pickup. The laser beam used on a CD-ROM player has a wavelength of

780 nano meters. DVD players employ a laser with a wavelength of 635 and 650 nano meters, designed to read through the thinner 0.6 mm transparent layer. This makes it possible to focus on smaller pits of digital data, about half the physical size of pits on a CD-ROM - effectively doubling the density of pits on a DVD-ROM. More data is squeezed onto the disc by recording tracks closer together and closer to the centre hole, as well as improving the error-correcting decoding algorithms.

DVD discs come in capacities of 4.7, 8.5, 9.4 and 17GB. Some of the early discs are single-sided, but the specification includes dual-layered and double-side versions that define the four levels of storage capacity. DVD data is read by a variable-focus laser; on dual-layered discs, a lens shifts the beam's focus from the pits on the outer layer to the pits on the inner layer. A comparative evaluation of the CD-ROM and DVD technologies is given in Table

Table 5 Comparison of CD-ROM and DVD Storage Characteristics

	CD-ROM Disc	*DVD Disc*
Disc diameter	120 mm	120 mm
Disc thickness	1.2 mm	1.2 mm
Centre hole diameter	15 mm	15 mm
Disc structure	Single substrate	Two bonded 0.6-mm substrates
Laser wavelength	780 nm	650 and 635 nm
Lens aperture	0.45	0.6
Track pitch	1.6 micron	0.74 micron
Shortest pit length	.834 micron	0.4 micron
Average bit rate	0.15 MB/s	4.7 MB/s
Data capacity	650-680 MB	Single-side/single-layer: 4.7 GB Single-side/dual-layer: 8.5 GB Double-side/single-layer: 9.4 GB Double-side/dual-layer: 17 GB
Data layers	1	1 or 2
Video compression	MPEG-1	MPEG-2
Audio compression	MPEG-1	Dolby Digital

Currently there are three different types of DVD drives that have been defined:

DVD-ROM: These are the drives with only the reading capability. They are used basically for removable mass storage for large volumes of data such as encyclopedia, and are currently available.

DVD-R: These are drives with write-once capability. DVD-R drives are also called "write once, read many" (WORM) drives and are currently available. These are similar to the CD-R drives with WORM capability.

DVD-RAM: These are drives with both read and write capability. DVD-RAM drives are also called "write many, read many" (WMRM) drives. Unfortunately there is no agreed format in this category. As a result there are a number of different formats that are being pushed by the various groups in the DVD forum. What was approved by the forum is a phase-change design that can hold 2.6GB of data per side on single- or double-sided disks. The single-sided disks will come in removable cartridges, but to protect the sensitive recording layer, double-sided disks will be permanently mounted in cartridges.

Currently, there is no standardization in the DVD formats. Both the +R and -R are currently available, and there is not much of difference between the two formats. Many drives are available that can write in both the formats, so that the user has little to worry about in terms of the format and usability.

DVD+RW: This is supported by Hewlett-Packard, Philips and Sony. DVD+RW's single-layer phase-change disks have more capacity than DVD-RAM disks - 4.7 GB per side, and use a higher-density recording process. The DVD+RW format does not rely on cartridges to hold the disks.

DVD-R/W: This is put forward by Pioneer and the first one to be available commercially. It will use random-access media that hold up to 4.7 GB. One of this technology's key characteristics is that its phase-change media have a higher reflectivity, and as a result, can be read in existing DVD-ROM drives and DVD players without modification.

SOFTWARE

Software represents that segment of the computing system, which determines the way the computer is to be used. Better software makes for a better utilization of the computing system. Various software items, which form part of any computing system, are shown in Fig. 13

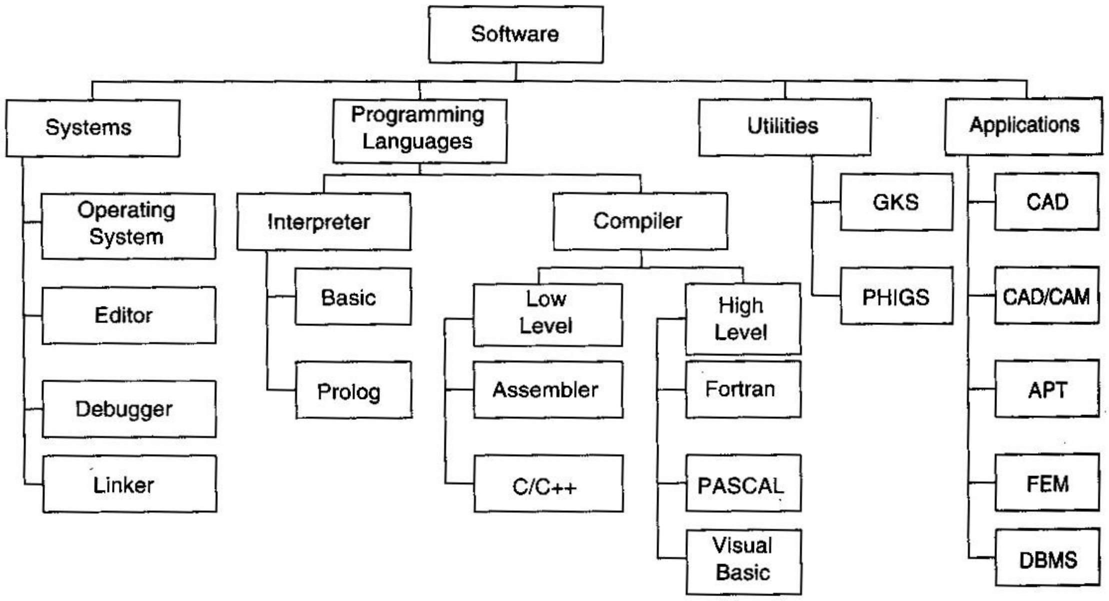


Fig. 13 - Software Classification

1) System Software

This represents the essential part of the software without which no computer system can operate. The operating system generally forms part of the hardware, and together with it provides for the use of all the hardware elements in an optimal manner. The operating systems are generally proprietary with the hardware that is being used. Examples are, VM for IBM computers, VMS for DEC computers, CP/M, PC-DOS (MSDOS), Windows 95 and Windows 98 for micro computers, Windows NT, UNIX and LINUX. UNIX and LINUX are the operating systems, which are generally considered independent of hardware.

The editors are meant, as the name indicates, for creating and modifying disc files; linker is to be used for linking the object modules that are developed into a coherent executable module; a debugger is to be used in program development to identify the logic and run-time errors.

2) Programming Languages

The next major segment of software is the programming languages, through which the software development takes place. Various languages have been developed to meet the different requirements of the applications. The programming languages are essentially translators and can be broadly classified into interpreters and compilers. Interpreter is a system in which the language is translated and then executed immediately. Thus, if the same statement is encountered a number of times during the execution of a given programme, it has to be translated on every occasion. However, in a compiler, the complete programme is translated once into the machine language and it can be executed any number of times. Interpreters are good for program development since the execution can take place immediately, but they are slow in view of the repeated translations that are needed. It is preferable to develop the program using an interpreter, and when it is bug-free it may be compiled to generate the directly executable machine language code for faster usage. The present trend in most of the programming languages is to provide an integrated environment consisting of an interpreter, compiler and context sensitive editor for the particular language. This would help in the rapid development of programs since no compilation and linking of the programs takes place during the development stage.

Traditionally, FORTRAN (Formula Translation) has been used as the programming language by the scientific community from the beginning. But now BASIC (Beginner's All purpose Symbolic Instruction Code), PASCAL and C are being used for CAD/CAM program development with C being the one used for the system development in view of its tight code and faster execution on various systems. With the artificial intelligence being increasingly used by the scientific community, LISP (List Processing) and PROLOG have also been used for some modules related to CAD/CAM.

Object oriented programming (OOP) has now become the norm of most of the programming. The objects are basically reusable code that can be used in many of the programs. By the careful designing of the objects, it is possible to reduce the system development time greatly, particularly for large programming projects.

3) Utilities

The utilities refer to a set of small programs which the applications developer can incorporate in his program for performing any specific task. Examples could be the numerical procedures, matrix operations, etc. These are essentially the many repeated applications which can once for all be made available as utilities and linked with any particular program, rather than trying to develop them anew every time they are needed.

4) Applications

Finally, these refer to the application programs, which are generally stand-alone programs that are meant for doing specific tasks. Examples are word processing, data base management, computer aided design, etc. In the later chapters, we will discuss more of these that relate to CAD/CAM.

Traditionally, computers have been classified as mainframe, mini computer and micro computers. The classification was based on the word length used, main memory available, and other such features. The developments in microprocessors that have taken place in the last few years have gradually decreased these differences with the lower end microcomputers acquiring the higher end facilities such as larger word length, higher memory addressable and so on.

The concept which is now gaining growth is the stand-alone work station which normally is a 32 or 64 bit microprocessor-based system having all the necessary hardware facilities at the local level (see Fig. 14). The microprocessor that is most popular in CAD/CAM workstations is the Intel Pentium family processors running up to 3 GHz. In addition a number of RISC processor based workstations are also widely used for CAD/CAM applications.

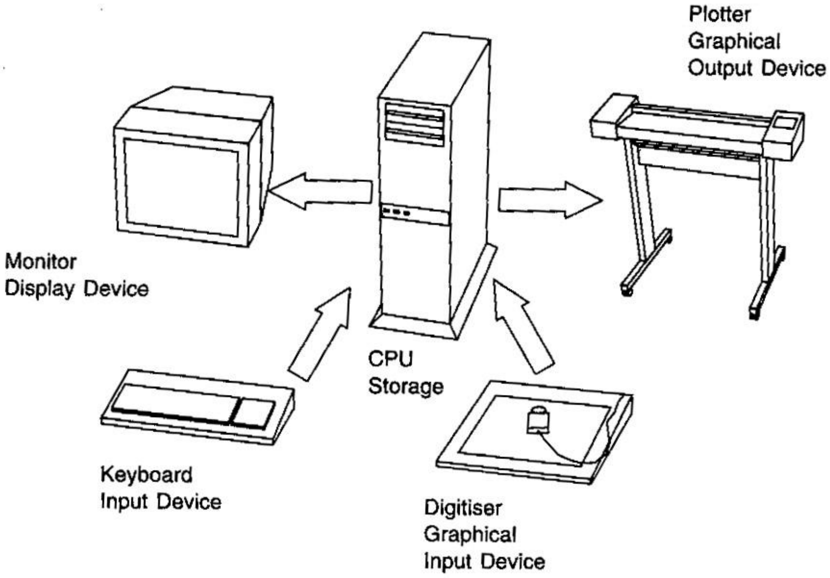


Fig. 14 Typical Configuration of a Workstation

The Intel Pentium based workstations are available at a low cost because of their large volume production. The majority of the CAD/CAM workstations as of today are based on Pentium or the RISC processors as described earlier. Based on the discussion in the earlier pages, we might be able to form the configuration of a suitable system based on the needs of CAD/CAM. A typical system for graphical applications may be as shown in **Fig. 14**.

QUESTIONS:

EXPERIMENT NO. - 02

AIM: To study about 2-D transformation.

The geometry traditionally followed is the Euclidean geometry. In the traditional sense we follow the Cartesian coordinate system specified by the X, Y and Z coordinate directions. The three axes are mutually perpendicular and would follow the right hand system.

In handling of geometrical information, many a times it becomes necessary to transform the geometry. The transformations actually convert the geometry from one coordinate system to the other.

The main types of pure transformations with which we are likely to come across are the following. These transformations are symbolically shown in Fig. 1.

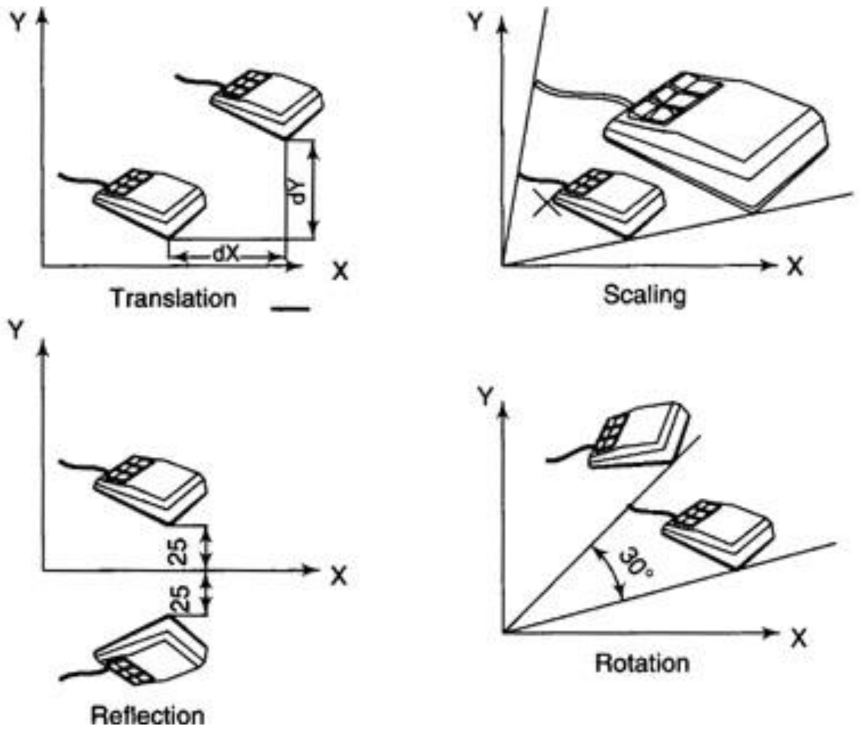


Fig. 1 Some of the Possible Geometric Transformations

- Translation
- Scaling
- Reflection or Mirror
- Rotation

A point in space can be represented by its coordinates (X, Y, Z) from the datum. As shown in Fig. 1 a point in 3 dimensions can be represented by the coordinates (X, Y, Z). The same can also be represented by a vector starting from the origin of the coordinate system as shown in Fig. 2.

In order to understand the system easily we would look at the transformations in 2 dimensional system for the sake of easy understanding. The same would then be extended to look at the 3 dimensional viewing.

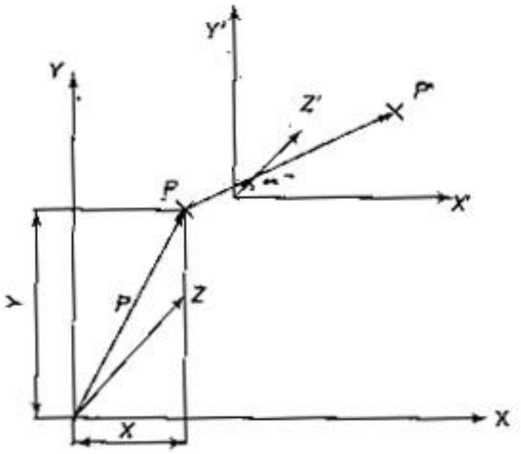


Fig. 2

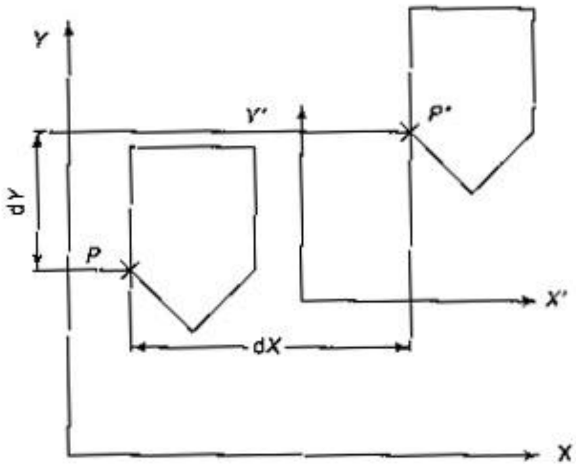


Fig. 3

3.4.1 Translation

It is the most common and easily understood transformation in CAD. This moves a geometric entity in space in such a way that the new entity is parallel at all points to the old entity. A representation is shown in Fig. 3 for an object. Let us now consider a point on the object, represented by P which is translated along X and Y-axes by dX and dY to a new position P*. The new coordinates after transformation are given by following equations.

$$P^* = [x^*, y^*] \tag{3.24}$$

$$x^* = x + dX \tag{3.25}$$

$$y^* = y + dY \tag{3.26}$$

Putting Eqs. 3.25 and 3.26 back into Eq. 3.24, we can write

$$[P^*] = \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} x + dX \\ y + dY \end{bmatrix} \tag{3.27}$$

This can also be written in matrix form as follows.

$$[P^*] = \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} x + dX \\ y + dY \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dX \\ dY \end{bmatrix} \tag{3.28}$$

This is normally the operation used in the CAD systems as MOVE command.

Scaling

Scaling is the transformation applied to change the scale of an entity. As shown in Fig. 4, this alters the size of the entity by the scaling factor applied. For example, in Fig. 4, to achieve scaling, the original coordinates would be multiplied uniformly by the scaling factor.

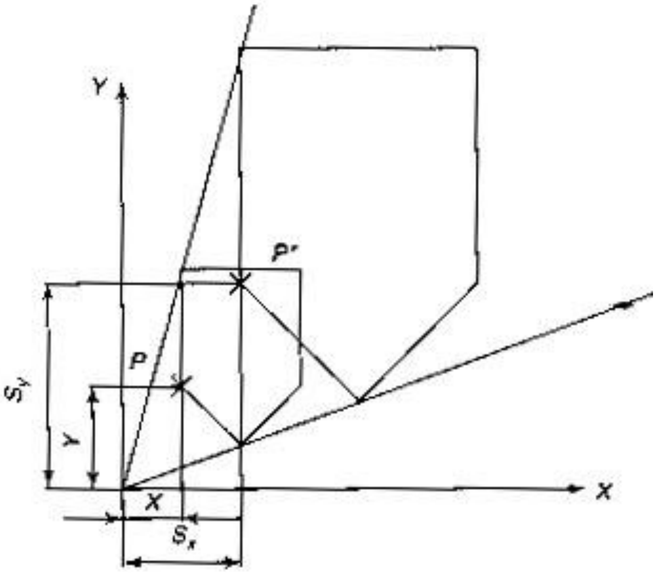


Fig. 4 Scaling of a Plane Figure

P* = [X*, Y*] = [S_x x X, S_y x Y]

This equation can also be represented in a matrix form as follows.

$$P^* = [X^*, Y^*] = [S_x \times X, S_y \times dY]$$

This equation can also be represented in a matrix form as follows.

$$[P^*] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$[P^*] = [T_s] \cdot [P]$$

where

$$[Ts] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

Since the scaling factors can be individually applied, there is a possibility to have differential scaling when $S_x \neq S_y$. Normally in the CAD systems uniform scaling is allowed for object manipulation. In the case of zoom facility in graphic systems, uniform scaling is applied. Zooming is only a display attribute and is applied only to the display and not to the actual geometric database.

Reflection or Mirror

Reflection or mirror is a transformation, which allows a copy of the object to be displayed while the object is reflected about a line or a plane. Typical examples are shown in Fig. 5, where in (a) shows reflection about X-axis, while the one in (b) shows the reflection about Y-axis. The reflection shown in (c) is about the X and Y-axis or about the origin.

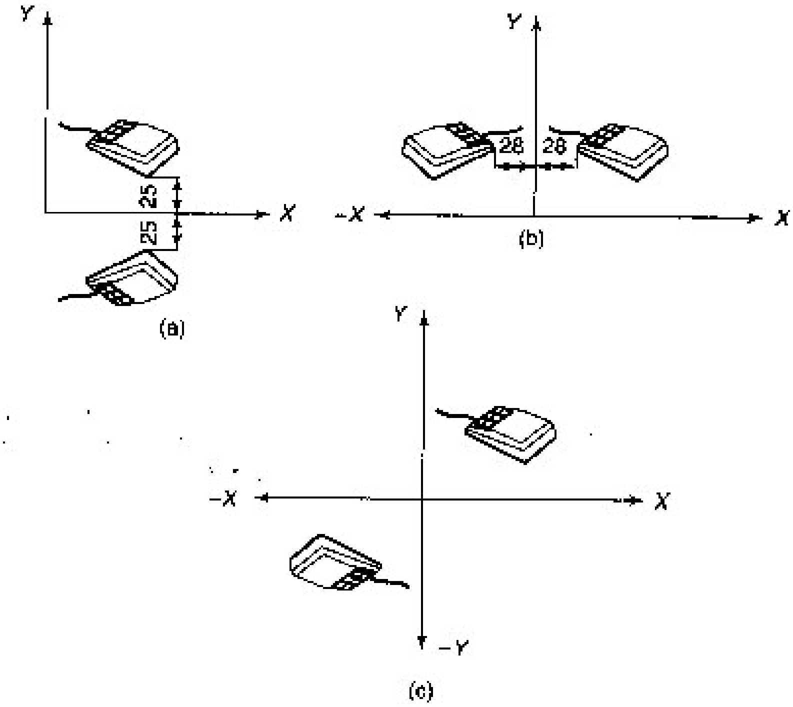


Fig. 5 Possible Reflection (Mirror) Transformations

The transformation required in this case is that the axes coordinates will get negated depending upon the reflection required. For example from Fig. 6, the new

P* = [X*, 7*] = [X, -Y] (3.33)

This can be given in a matrix form as

$$P^* = [X^*,\ Y^*] = [S_x \times X,\ S_y \times dY]$$

This equation can also be represented in a matrix form as follows.

$$[P^*] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$[P^*] = [T_s] \cdot [P]$$

where

$$[Ts] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

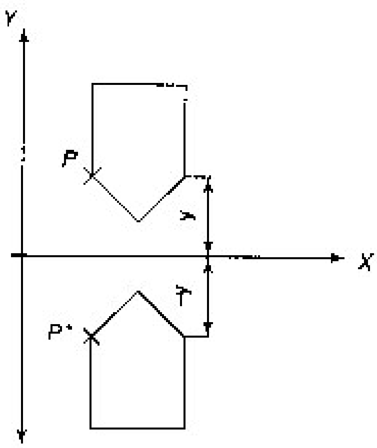


Fig. 6 Example for Reflection Transformation

Here, -1 in the first position refers to the reflection about Y-axis where all the X coordinate values get negated. When the second term becomes -1 the reflection will be about the X-axis with all Y coordinate values getting reversed. Both the values are -1 for reflection about X and Y-axes.

Rotation

Rotation is another important geometric transformation. The final position and orientation of a geometric entity is decided by the angle of rotation (0) and the base point about which the rotation is to be done (Fig. 7).

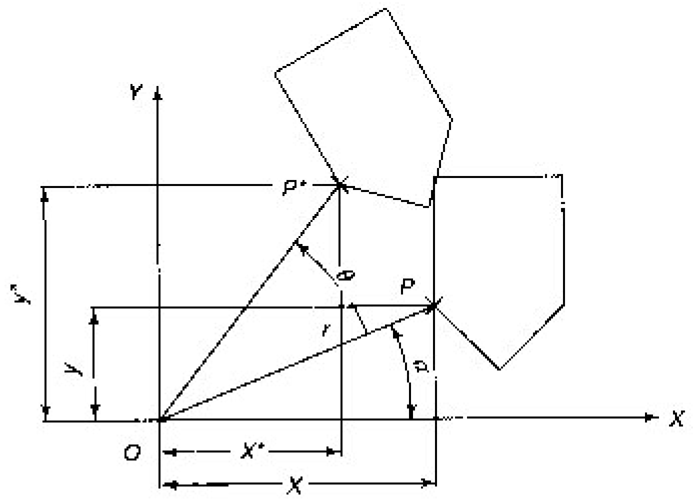


Fig. 7 Rotation Transformation

To develop the transformation matrix for transformation, consider a point P located in XY plane, being rotated in the counter clockwise direction to the new position, P* by an angle 6 as shown in Fig. 7. The new position P* is given by

P* = [x*, y*]

From the Fig. 8, the original position is specified by

$x = r \cos \alpha$

$y = r \sin \alpha$

The new position, P* is specified by

$x^* = r \cos (\alpha + \theta)$

$= r \cos \theta \cos \alpha - r \sin \theta \ \sin \alpha$

$= x \cos \theta - y \sin \theta$

$y^* = r \sin (\alpha + \theta)$

$= r \sin \theta \cos \alpha + r \cos \theta \sin \alpha = x \sin \theta + y \cos \theta$

This can be written in a matrix form as

$$[T_R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3.38}$$

The above is the transformation matrix for rotation which can be applied in any plane in the following way.

$$\begin{bmatrix} y^* \\ z^* \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \tag{3.39}$$

$$\begin{bmatrix} z^* \\ x^* \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} \tag{3.40}$$

Homogeneous Representation

In order to concatenate the transformations as shown in Eq. 3.41, all the transformation matrices should be multiplicative type. However, as seen earlier, the translation matrix (Eq. 3.28) is vector additive while all others are matrix multiplications. The following form could be used to convert the translation into a multiplication form.

$$[P^*] = \begin{bmatrix} x^* \\ y^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dX \\ 0 & 1 & dY \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{3.42}$$

Hence the translation matrix in multiplication form can be given as

$$[MT] = \begin{bmatrix} 1 & 0 & dX \\ 0 & 1 & dY \\ 0 & 0 & 1 \end{bmatrix} \tag{3.43}$$

This is termed as homogeneous representation. In homogeneous representation, an n-dimensional space is mapped into (n + 1) dimensional space. Thus a 2 dimensional point [x y] is represented by 3 dimensions as [x y 1]. This greatly facilitates the computer graphics operations where the concatenation | of multiple transformations can be easily carried out. This can be experienced in the following situations.

$$[T1] = \begin{bmatrix} 1 & 0 & -dX \\ 0 & 1 & -dY \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T2] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T3] = \begin{bmatrix} 1 & 0 & dX \\ 0 & 1 & dY \\ 0 & 0 & 1 \end{bmatrix}$$

The required transformation matrix is given by

$$[T] = [T3]\,[T2]\,[T1]$$

$$[T] = \begin{bmatrix} 1 & 0 & dX \\ 0 & 1 & dY \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -dX \\ 0 & 1 & -dY \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta & dX(1-\cos\theta) + dY\sin\theta \\ \sin\theta & \cos\theta & -dX\sin\theta + dY(1-\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

3D TRANSFORMATION

The 2D transformations as explained in earlier sections could be extended to 3D by adding the Z-axis parameter. The transformation matrix will now be 4 X 4. The following are the transformation matrices to be used for this purpose.

Translation

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & dX \\ 0 & 1 & 0 & dY \\ 0 & 0 & 1 & dZ \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.46)$$

Scaling

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.47)$$

Reflection

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} \pm 1 & 0 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.48)$$

Rotation about Z-axis (XY plane)

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.49)$$

Rotation about X-axis (YZ Plane)

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \sin\theta & -\cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.50)$$

Rotation about Y-axis (ZX Plane)

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & -\cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (3.51)$$

Ref.: CAD/CAM Principles and Applications by P.N. Rao.

QUESTIONS:

EXPERIMENT NO. - 03

AIM: To study the optimum design method and to solve problem using optimum design method on optimum design of mechanical element.

INTRODUCTION

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function. It can be seen from Fig. 1.1 that if a point x* corresponds to the minimum value of function f (x), the same point also corresponds to the maximum value of the negative of the function, f (x). Thus, without loss of generality, optimization can be taken to mean minimization since the maximum of a function can be found by seeking the minimum of the negative of the same function. There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems.

The optimum seeking methods are also known as mathematical programming techniques and are generally studied as a part of operations research. Operations research is a branch of mathematics concerned with the application of scientific methods and techniques to decision-making problems and with establishing the best or optimal solutions. Table 1.1 lists various mathematical programming techniques together with other well-defined areas of operations research. The classification given in Table 1.1 is not unique; it is given mainly for convenience.

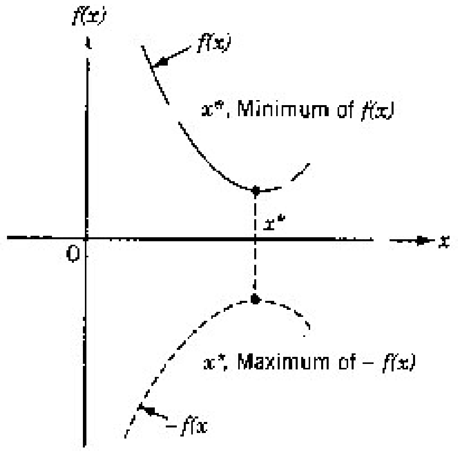


Figure 1 Minimum of f (x) is same as maximum of — f (x).

Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques can be used to analyze problems described by a set of random variables having known probability distributions. Statistical methods enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation. This book deals with the theory and application of mathematical programming techniques suitable for the solution of engineering design problems.

ENGINEERING APPLICATIONS OF OPTIMIZATION

Optimization, in its broadest sense, can be applied to solve any engineering problem. To indicate the wide scope of the subject, some typical applications from different engineering disciplines are given below.

1. Design of aircraft and aerospace structures for minimum weight
2. Finding the optimal trajectories of space vehicles
3. Design of civil engineering structures such as frames, foundations, bridges, towers, chimneys, and dams for minimum cost
4. Minimum-weight design of structures for earthquake, wind, and other types of random loading
5. Design of water resources systems for maximum benefit
6. Optimal plastic design of structures
7. Optimum design of linkages, cams, gears, machine tools, and other mechanical components
8. Selection of machining conditions in metal cutting processes for minimum production cost
9. Design of material handling equipment such as conveyors, trucks, and cranes for minimum cost
10. Design of pumps, turbines, and heat transfer equipment for maximum efficiency
11. Optimum design of electrical machinery such as motors, generators, and transformers
12. Optimum design of electrical networks
13. Shortest route taken by a salesperson visiting various cities during one tour
14. Optimal production planning, controlling, and scheduling
15. Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of the physical phenomenon
16. Optimum design of chemical processing equipment and plants
17. Design of optimum pipeline networks for process industries
18. Selection of a site for an industry
19. Planning of maintenance and replacement of equipment to reduce operating costs
20. Inventory control
21. Allocation of resources or services among several activities to maximize the benefit
22. Controlling the waiting and idle times and queuing in production lines to reduce the costs
23. Planning the best strategy to obtain maximum profit in the presence of a competitor
24. Optimum design of control systems

STATEMENT OF AN OPTIMIZATION PROBLEM

An optimization or a mathematical programming problem can be stated as follows.

Find X = **Which minimizes f(X)** **(1.1)**

Subject to the constraints

$$g_j(X) \le 0, \quad j = 1,2,\ldots,m$$
$$l_j(X) = 0, \quad j = 1,2,\ldots,p$$

Where X is an n-dimensional vector called the design vector, /(X) is termed the objective Junction, and g, (X) and /, (X) are known as inequality and equality constraints, respectively. The number of variables n and the number of constraints m and/or p need not be related in any way. The problem stated in Eq. (1.1) is called a constrained optimization problem." Some optimization problems do not involve any constraints and can be stated as:

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ which minimizes $f(X)$ (1.2)

Such problems are called unconstrained optimization problems.

Design Vector

Any engineering system or component is denned by a set of quantities some of which are viewed as variables during the design process. In general, certain quantities are usually fixed at the outset and these are called pre-assigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables x_i, i = 1, 2, . . . n. The design variables are collectively represented as a design vector-

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

As an example, consider the design of the gear pair shown in Fig. 2, characterized by its face width b, number of teeth T_1 and T_2, center distance d, pressure angle ψ, tooth profile, and material. If center distance d, pressure angle ψ, tooth profile, and material of the gears are fixed in advance, these quantities can be called pre-assigned parameters. The remaining quantities can be collectively represented by a design vector

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b \\ T_1 \\ T_2 \end{Bmatrix}$$

If there are no restrictions on the choice of b, T_1, and T_2, any set of three numbers will constitute a design for the gear pair. If an n-dimensional Cartesian space with each coordinate axis representing a design variable x_i (i = 1, 2 . . . n) is considered, the space is called the design variable space or simply, design space. Each point in the n-dimensional design space is called a design point and represents either a possible or an impossible solution to the design problem. In the case of the design of a gear pair, the design point

$$\begin{Bmatrix} 1.0 \\ 20 \\ 40 \end{Bmatrix},$$

for example, represents a possible solution, whereas the design point

$$\begin{Bmatrix} 1.0 \\ -20 \\ 40.5 \end{Bmatrix}$$

represents an impossible solution since it is not possible to have either a negative value or a fractional value for the number of teeth.

Design Constraints

In many practical problems, the design variables cannot be chosen arbitrarily; rather, they have to satisfy certain specified functional and other requirements. The restrictions that must be satisfied to produce an acceptable design are collectively called design constraints. Constraints that represent limitations on the behavior or performance of the system are termed behavior or functional constraints. Constraints that represent physical limitations on design variables such as availability, fabricability, and transportability are known as geometric or side constraints. For example, for the gear pair shown in Fig. 1.2, the face width b cannot be taken smaller than a certain value, due to strength requirements. Similarly, the ratio of the numbers of teeth, T_1/T_2, is dictated by the speeds of the input and output shafts, N_1 and N_2. Since these constraints depend on the performance of the gear pair, they are called behavior constraints. The values of T_1 and T_2 cannot be any real numbers but can only be integers. Further, there can be upper and lower bounds on T_1 and T_2 due to manufacturing limitations. Since these constraints depend on the physical limitations, they are called side constraints.

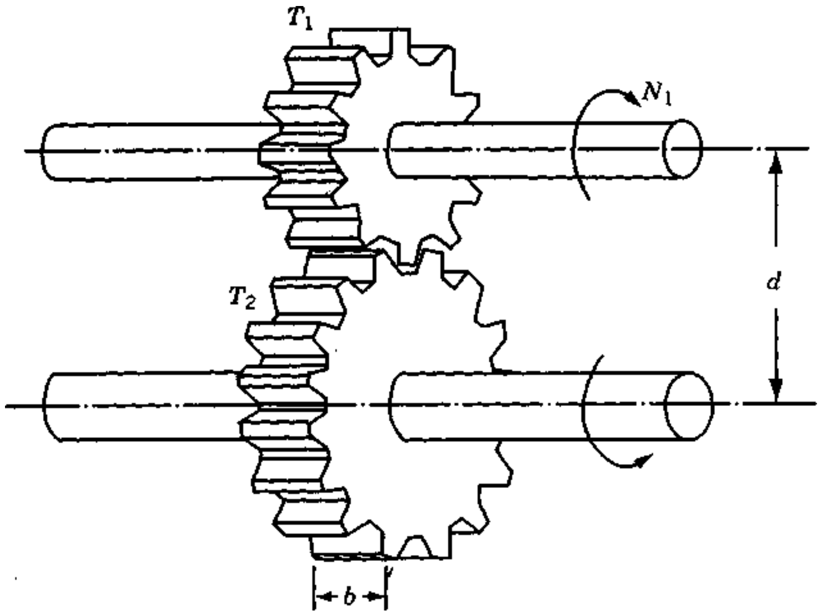


Figure 2 Gear pair in mesh.

Constraint Surface

For illustration, consider an optimization problem with only inequality constraints $g_j(X) \leq 0$. The set of values of X that satisfy the equation $g_j(X) = 0$ forms a hyper surface in the design space and is called a constraint surface. Note that this is an (n – l)-dimensional subspace, where n is the number of design variables. The constraint surface divides the design space into two regions: one in which $g_j(X) < 0$ and the other in which $g_j(X) > 0$. Thus the points lying on the hyper surface will satisfy the constraint $g_i(X)$ critically, whereas the points lying in the region where $g_j(X) > 0$ are infeasible or unacceptable, and the points lying in the region where $g_j(X) < 0$ are feasible or acceptable. The collection of all the constraint surfaces $g_j(X) = 0$, $j = 1,2, . . .m$, which separates the acceptable region is called the composite constraint surface.

Figure 3 shows a hypothetical two-dimensional design space where the infeasible region is indicated by hatched lines. A design point that lies on one or more than one constraint surface is called a bound point, and the associated constraint is called an active constraint. Design points that do not lie on any constraint surface are known as free points. Depending on whether a particular design point belongs to the acceptable or unacceptable region, it can be identified as one of the following four types:

1. Free and acceptable point
2. Free and unacceptable point
3. Bound and acceptable point
4. Bound and unacceptable point

All four types of points are shown in Fig. 3.

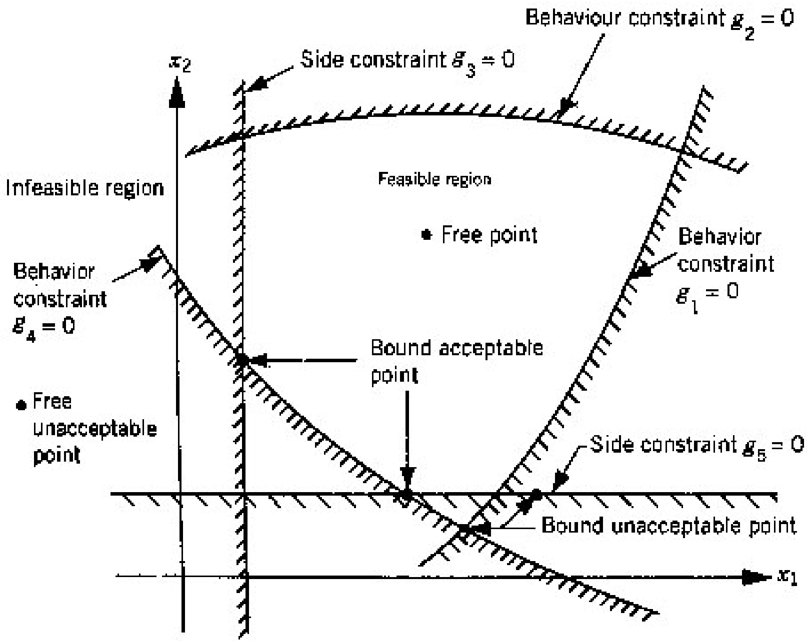


Figure 3 Constraint surfaces in a hypothetical two-dimensional design space.

Objective Function

The conventional design procedures aim at finding an acceptable or adequate design, which merely satisfies the functional and other requirements of the problem. In general, there will be more than one acceptable design, and the purpose of optimization is to choose the best one of the many acceptable designs available. Thus a criterion has to be chosen for comparing the different alternative acceptable designs and for selecting the best one. The criterion, with respect to which the design is optimized, when expressed as a function of the design variables, is known as the criterion or merit or objective function. The choice of objective function is governed by the nature of problem: The objective function for minimization is generally taken as weight in aircraft and aerospace structural design problems. In civil engineering structural designs, the objective is usually taken as the minimization of cost. The maximization of mechanical efficiency is the obvious choice of an objective in mechanical engineering systems design. Thus the choice of the objective function appears to be straightforward in most design problems. However, there may be cases where the optimization with respect to a particular criterion may lead to results that may not be satisfactory with respect to another criterion. For example, in mechanical design, a gearbox transmitting the maximum power may not have the minimum weight. Similarly, in structural design, the minimum-weight design may not correspond to minimum stress design, and the minimum stress design, again, may not correspond to maximum frequency design. Thus the selection of the objective function can be one of the most important decisions in the whole optimum design process.

In some situations, there may be more than one criterion to be satisfied simultaneously. For example, a gear pair may have to be designed for minimum weight and maximum efficiency while transmitting a specified horsepower. An optimization problem involving multiple objective functions is known as a multiobjective-programming problem. With multiple objectives there arises a possibility of conflict, and one simple way to handle the problem is to construct an overall objective function as a linear combination of the conflicting multiple objective functions. Thus if $f_1(X)$ and$f_2(X)$ denote two objective functions, construct a new (overall) objective function for optimization as

$$f(X) = \alpha_1 f_1(X) + \alpha_2 f_2(X)$$

CLASSIFICATION OF OPTIMIZATION PROBLEMS

Optimization problems can be classified in several ways, as described below.

1) Classification Based on the Existence of Constraints

As indicated earlier, any optimization problem can be classified as constrained or unconstrained, depending on whether or not constraints exist in the problem.

2) Classification Based on the Nature of the Design Variables

Based on the nature of design variables encountered, optimization problems can be classified into two broad categories. In the first category, the problem is to find values to a set of design parameters that make some prescribed function of these parameters minimum subject to certain constraints. For example, the problem of minimum-weight design of a prismatic beam shown in Fig. 4a subject to a limitation on the maximum deflection can be stated as follows.

$$\text{Find } \mathbf{X} = \begin{Bmatrix} b \\ d \end{Bmatrix} \text{ which minimizes} \tag{1.4}$$

$$f(\mathbf{X}) = \rho l b d$$

subject to the constraints

$$\delta_{tip}(\mathbf{X}) \leq \delta_{max}$$

$$b \geq 0$$

$$d \geq 0$$

Where p is the density and δtip is the tip deflection of the beam. Such problems are called parameter or static optimization problems. In the second category of problems, the objective is to find a set of design parameters, which are all continuous functions of some other parameter that minimizes an objective function subject to a set of constraints. If the cross-sectional dimensions of the rectangular beam are allowed to vary along its length as shown in Fig. 4b, the optimization problem can be stated as:

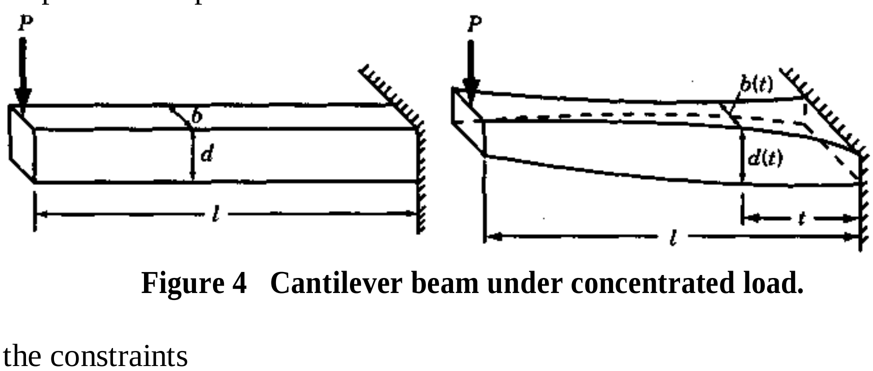


Figure 4 Cantilever beam under concentrated load.

subject to the constraints

$$\delta_{tip}[X(t)] \leq \delta_{max}, \quad 0 \leq t \leq l$$

$$b(t) \geq 0, \quad 0 \leq t \leq l$$

$$d(t) \geq 0, \quad 0 \leq t \leq l$$

Here the design variables are functions of the length parameter t. This type of problem, where each design variable is a function of one or more parameters is known as a trajectory or dynamic optimization problem.

3) Classification Based on the Physical Structure of the Problem

Depending on the physical structure of the problem, optimization problems can be classified as optimal control and no optimal control problems.

Optimal Control Problem: An optimal control (OC) problem is a mathematical programming problem involving a number of stages, where each stage evolves from the preceding stage in a prescribed manner. It is usually described by two types of variables: the control (design) and the state variables. The control variables define the system and govern the evolution of the system from one stage to the next, and the state variables describe the behavior or status of the system in any stage. The problem is to find a set of control or design variables such that the total objective function (also known as the performance index, PI) over all the stages is minimized subject to a set of constraints on the control and state variables. An OC problem can be stated as follows:

$$\text{Find } \mathbf{X} \text{ which minimizes } f(\mathbf{X}) = \sum_{i=1}^{l} f_i(x_i, y_i) \qquad (1.6)$$

Subject to the constraints

$$q_i(x_i, y_i) + y_i = y_i + 1, \qquad i = 1,2,\ldots,l$$
$$g_j(x_j) \le 0, \qquad j = 1,2,\ldots,l$$
$$h_k(y_k) \le 0, \qquad k = 1,2,\ldots,l$$

Where x_i, is the i^{th} control variable, y_i the i^{th} state variable, and f_i the contribution of the i^{th} stage to the total objective function; g_j, h_k, and q_i, are functions of x_j, y_k and x_i, and y_i, respectively, and l is the total number of stages. The control and state variables x_i and y_i can be vectors in some cases.

4) Classification Based on the Nature of the Equations Involved

Another important classification of optimization problems is based on the nature of expressions for the objective function and the constraints. According to this classification, optimization problems can be classified as linear, nonlinear, geometric, and quadratic programming problems. This classification is extremely useful from the computational point of view since there are many special methods available for the efficient solution of a particular class of problems. Thus the first task of a designer would be to investigate the class of problem encountered. This will, in many cases, dictate the types of solution procedures to be adopted in solving the problem.

Nonlinear Programming Problem: If any of the functions between the objective and constraint functions in Eq. (1.1) is nonlinear; the problem is called a nonlinear programming (NLP) problem. This is the most general programming problem and all other problems can be considered as special cases of the NLP problem.

Geometric Programming Problem

Definition: A function h(X) is called a posynomial if h can be expressed as the sum of power terms each of the form

$$c_i x_1^{ai1} x_2^{ai2} \ldots x_n^{ain}$$

where c_i, and c_{ij} are constants with $c_i > 0$ and $x_j > 0$. Thus a posynomial with N terms can be expressed as

$$h(X) = c_i x_1^{ai1} x_2^{ai2} \ldots x_n^{ain} + \ldots + c_N x_1^{aN1} x_2^{aN2} \ldots x_n^{aNn} \qquad (1.7)$$

A geometric programming (GMP) problem is one in which the objective function and constraints are expressed as posynomials in X. Thus GMP problem can be posed as follows [1.44]:

Find X which minimizes

$$f(\mathbf{X}) = \sum_{i=1}^{N_0} c_i \left(\prod_{j=1}^{n} x_j^{p_{ij}} \right), \quad c_i > 0, \quad x_j > 0 \tag{1.8}$$

subject to

$$g_k(\mathbf{X}) = \sum_{i=1}^{N_k} a_{ik} \left(\prod_{j=1}^{n} x_j^{q_{ijk}} \right) > 0, \quad a_{ik} > 0, \quad x_j > 0, \quad k = 1,2,\ldots,m$$

Where N_o and N_k denote the number of posynomial terms in the objective and kth constraint function, respectively.

Quadratic Programming Problem: A quadratic programming problem is a nonlinear programming problem with a quadratic objective function and linear constraints. It is usually formulated as follows:

$$F(\mathbf{X}) = c + \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} x_i x_j \tag{1.9}$$

subject to

$$\sum_{i=1}^{n} a_{ij} x_i = b_j, \quad j = 1,2,\ldots,m$$

$$x_i \geq 0, \quad i = 1,2,\ldots,n$$

Where c, q_i, Q_{ij}, a_{ij}, and b_j, are constant

Linear Programming Problem: If the objective function and all the constraints in Eq. (1.1) are linear functions of the design variables, the mathematical programming problem is called a linear programming (LP) problem. A linear programming problem is often stated in the following standard form:

$$\textbf{Find X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

$$\text{which minimizes } f(\mathbf{X}) = \sum_{i=1}^{n} c_i x_i$$

Subject to the constraints

where c_i, a_{ij}, and b_j are constants.

Classification Based on the Permissible Values of the Design Variables

Depending on the values permitted for the design variables, optimization problems can be classified as integer- and real-valued programming problems.

Integer Programming Problem: If some or all of the design variables x\, x_2, ... , x_n of an optimization problem are restricted to take on only integer (or discrete) values, the problem is called an integer programming problem. On the other hand, if all the design variables are permitted to take any real value, the optimization problem is called a real-valued programming problem. According to this definition, the problems considered in Examples are real-valued programming problems.

Classification Based on the Deterministic Nature of the Variables

Based on the deterministic nature of the variables involved, optimization problems can be classified as deterministic and stochastic programming problems.

Stochastic Programming Problem: A stochastic programming problem is an optimization problem in which some or all of the parameters (design variables and/or pre-assigned parameters) are probabilistic (nondeterministic or stochastic). According to this definition, the problems considered in Examples 1.1 to 1.7 are deterministic programming problems.

Classification Based on the Separability of the Functions

Optimization problems can be classified as separable and nonseparable programming problems based on the separability of the objective and constraint functions.

Separable Programming Problem:
Definition: A function f(X) is said to be separable if it can be expressed as the sum of n single-variable functions, $f_t(x_l)$,$f_2(x_2)$, . . . ,$f_n(x_n)$, that is,

$$f(\mathbf{X}) = \sum_{i=1}^{n} f_i(x_i) \qquad (1.11)$$

A separable programming problem is one in which the objective function and the constraints are separable and can be expressed in standard form as:

$$\text{Find } \mathbf{X} \text{ which minimizes } f(\mathbf{X}) = \sum_{i=1}^{n} f_i(x_i) \qquad (1.12)$$

subject to

$$g_j(\mathbf{X}) = \sum_{i=1}^{n} g_{ij}(x_i) \le b_j, \qquad j = 1,2,\ldots,m$$

where b_j is a constant.

Classification Based on the Number of Objective Functions

Depending on the number of objective functions to be minimized, optimization problems can be classified as single- and multi objective programming problems. According to this classification, the problems considered in Examples 1.1 to 1.9 are single objective programming problems.

Multi Objective Programming Problem: A multi objective programming problem can be stated as follows:

Find X which minimizes $f_1(X)$,$f_2(X)$,. . .,$f_k(X)$

Subject to

$g_j(X) \le 0, \quad j = 1,2,...,m$

where f_1, f_2,. . ., f_k denote the objective functions to be minimized simultaneously.

REF.: Engineering Optimization – by S.S. Rao.

QUESTIONS:

EXPERIMENT NO.- 04

AIM: To study the finite element analysis and it's application to solving mechanical engineering problems.

BASIC CONCEPT

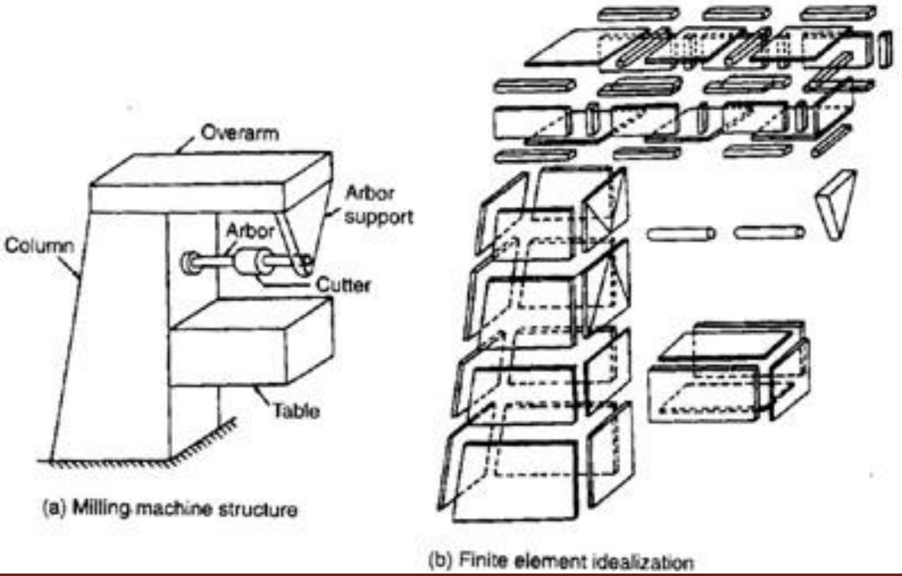
The basic idea in the finite element method is to find the solution of a complicated problem by replacing it by a simpler one. Since a simpler one in finding the solution replaces the actual problem, we will be able to find only an approximate solution rather than the exact solution. The existing mathematical tools will not be sufficient to find the exact solution (and sometimes, even an approximate solution) of most of the practical problems. Thus, in the absence of any other convenient method to find even the approximate solution of a given problem, we have to prefer the finite element method. Moreover, in the finite element method, it will often be possible to improve or refine the approximate solution by spending more computational effort.

In the finite element method, the solution region is considered as built up of many small, interconnected sub regions called finite elements. As an example of how a finite element model might be used to represent a complex geometrical shape, consider the milling machine structure shown in Figure 1.1(a). Since it is very difficult to find the exact response (like stresses and displacements) of the machine under any specified cutting (loading) condition, this structure is approximated as composed of several pieces as shown in Fig. l.l(b) in the finite element method. In each piece or element, a convenient approximate solution is assumed and the conditions of overall equilibrium of the structure are derived. The satisfaction of these conditions will yield an approximate solution for the displacements and stresses. Figure 1.2 shows the finite element idealization of a fighter aircraft.

HISTORICAL BACKGROUND

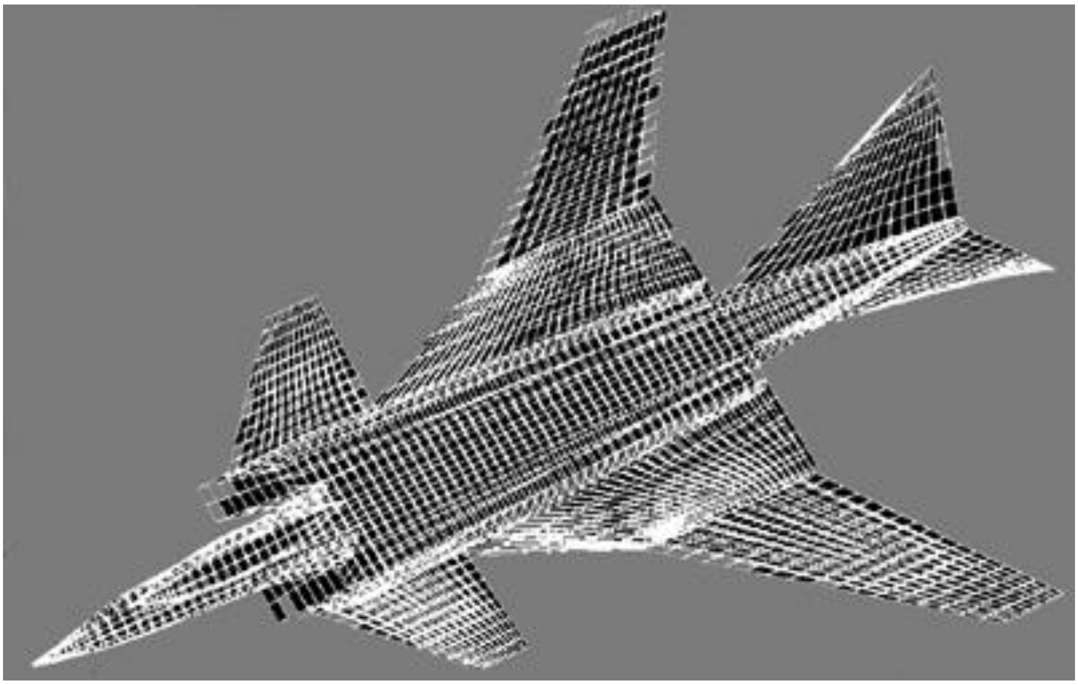
Although the name of the finite element method was given recently, the concept dates back for several centuries. For example, ancient mathematicians found the circumference of a circle by approximating it by the perimeter of a polygon as shown in Figure 1.3.

In terms of the present-day notation, each side of the polygon can be called a "finite element." By considering the approximating polygon inscribed or circumscribed, one can obtain a lower bound $S^{(l)}$ or an upper bound $S^{(u)}$ for the true circumference *S.* Furthermore, as the number of sides of the polygon is increased, the approximate values converge to the true value.



(a) Milling machine structure

(b) Finite element idealization



ENGINEERING APPLICATIONS OF THE FINITE ELEMENT METHOD

As stated earlier, the finite element method was developed originally for the analysis aircraft structures. However, the general nature of its theory makes it applicable to wide variety of boundary value problems in engineering. A boundary value problem is one in which a solution is sought in the domain (or region) of a body subject to the satisfaction of prescribed boundary (edge) conditions on the dependent variables or their derivatives. Table 1.1 gives specific applications of the finite element method in the three major categories of boundary value problems, namely, (i) equilibrium or steady-state or time-independent problems, (ii) eigenvalue problems, and (iii) propagation or transient problems.

In an equilibrium problem, we need to find the steady-state displacement or stress distribution if it is a solid mechanics problem, temperature or heat flux distribution if it is a heat transfer problem, and pressure or velocity distribution if it is a fluid mechanic problem.

In eigenvalue problems also, time will not appear explicitly. They may be considered as extensions of equilibrium problems in which critical values of certain parameters are to be determined in addition to the corresponding steady-state configurations. In these problems, we need to find the natural frequencies or buckling loads and mode shapes if it is a solid mechanics or structures problem, stability of laminar flows if it is a fluid mechanic problem, and resonance characteristics if it is an electrical circuit problem.

The propagation or transient problems are time-dependent problems. This type of problem arises, for example, whenever we are interested in finding the response of a body under time-varying force in the area of solid mechanics and under sudden heating or cooling in the field of heat transfer.

GENERAL DESCRIPTION OF THE FINITE ELEMENT METHOD

In the finite element method, the actual continuum or body of matter, such as a solid liquid, or gas, is represented as an assemblage of subdivisions called finite elements. These elements are considered to be interconnected at specified joints called nodes or nodal points. The nodes usually lie on the element boundaries where adjacent elements are considered to be connected. Since the actual variation of the field variable (e.g., displacement stress, temperature, pressure, or velocity) inside the continuum is not known, we assume that the variation of the field variable inside a finite element can be approximated by a simple function. These approximating functions (also called interpolation models) are defined in terms of the values of the field variables at the nodes. When field equations (like equilibrium equations) for the whole continuum are written, the new unknowns will be the nodal values of the field variable. By solving the field equations, which are generally in the form of matrix equations, the nodal values of the field variable will be known. Once these are known, the approximating functions define the field variable throughout the assemblage of elements.

The solution of a general continuum problem by the finite element method always follows an orderly step-by-step process. With reference to static structural problems, the step-by-step procedure can be stated as follows:

Table 1.1. Engineering Applications of the Finite Element Method

Area of study	Equilibrium problems	Eigenvalue problems	Propagation problems
1. Civil engineering structures	Static analysis of trusses, frames, folded plates, shell roofs, shear walls, bridges, and prestressed concrete structures	Natural frequencies and modes of structures; stability of structures	Propagation of stress waves; response of structures to aperiodic loads
2. Aircraft structures	Static analysis of aircraft wings, fuselages, fins, rockets, spacecraft, and missile structures	Natural frequencies, flutter, and stability of aircraft, rocket, spacecraft, and missile structures	Response of aircraft structures to random loads; dynamic response of aircraft and spacecraft to aperiodic loads
3. Heat conduction	Steady-state temperature distribution in solids and fluids	—	Transient heat flow in rocket nozzles, internal combustion engines, turbine blades, fins, and building structures
4. Geomechanics	Analysis of excavations, retaining walls, underground openings, rock joints and soil–structure interaction problems; stress analysis in soils, dams, layered piles, and machine foundations	Natural frequencies and modes of dam–reservoir systems and soil–structure interaction problems	Time-dependent soil–structure interaction problems; transient seepage in soils and rocks; stress wave propagation in soils and rocks
5. Hydraulic and water resources engineering; hydrodynamics	Analysis of potential flows, free surface flows, boundary layer flows, viscous flows, transonic aerodynamic problems; analysis of hydraulic structures and dams	Natural periods and modes of shallow basins, lakes, and harbors; sloshing of liquids in rigid and flexible containers	Analysis of unsteady fluid flow and wave propagation problems; transient seepage in aquifers and porous media; rarefied gas dynamics; magneto-hydrodynamic flows

(continued)

Step (i): Discretization of the structure

The first step in the finite element method is to divide the structure or solution region into subdivisions or elements. Hence, the structure is to be modeled with suitable finite elements. The number, type, size, and arrangement of the elements are to be decided.

Step (ii): Selection of a proper interpolation or displacement model

Since the displacement solution of a complex structure under any specified load conditions cannot be predicted exactly, we assume some suitable solution within an element to approximate the unknown solution. The assumed solution must be simple from a computational standpoint, but it should satisfy certain convergence requirements. In general, the solution or the interpolation model is taken in the form of a polynomial.

Step (iii): Derivation of element stiffness matrices and load vectors

Prom the assumed displacement model, the stiffness matrix $[K^{(e)}]$ and the load vector $P^{(e)}$ of element e are to be derived by using either equilibrium conditions or a suitable variational principle.

Step (iv): Assemblage of element equations to obtain the overall equilibrium equations

Since the structure is composed of several finite elements, the individual element stiffness matrices and load vectors are to be assembled in a suitable manner and the overall equilibrium equations have to be formulated as

$$[K]\ \underline{\vec{\Phi}} = \underline{\vec{P}}$$

Where [K] is the assembled stiffness matrix, $\underline{\vec{\Phi}}$ is the vector of nodal displacements, and $\underline{\vec{P}}$ is the vector of nodal forces for the complete structure.

Step (v): Solution for the unknown nodal displacements

The overall equilibrium equations have to be modified to account for the boundary conditions of the problem. After the incorporation of the boundary conditions, the equilibrium equations can be expressed as

$$[K]\ \vec{\Phi} = \vec{P}$$

For linear problems, the vector $\vec{\Phi}$ can be solved very easily. However, for nonlinear problems, the solution has to be obtained in a sequence of steps, with each step involving the modification of the stiffness matrix [K] and/or the load vector P.

Step (vi): Computation of element strains and stresses

From the known nodal displacements *$*, if required, the element strains and stresses can be computed by using the necessary equations of solid or structural mechanics.

The terminology used in the previous six steps has to be modified if we want to extend the concept to other fields. For example, we have to use the term continuum or domain in place of structure, field variable in place of displacement, characteristic matrix in place of stiffness matrix, and element resultants in place of element strains. The application of the six steps of the finite element analysis is illustrated with the help of the following examples.

<u>DISCRETIZATION OF THE DOMAIN</u>

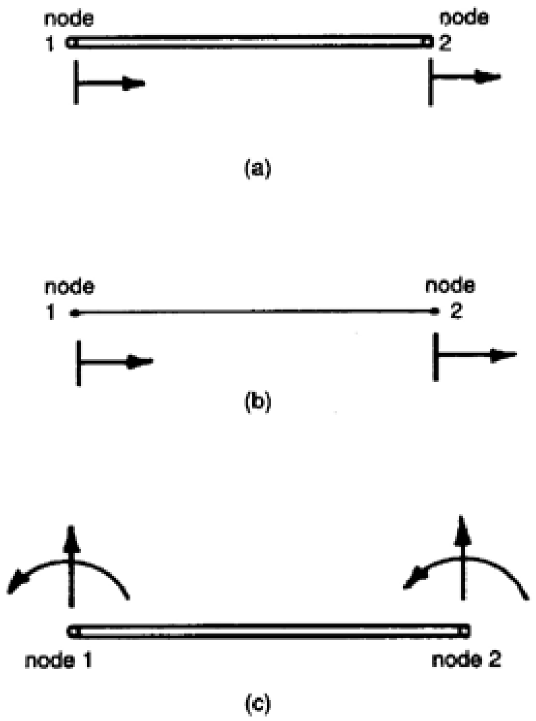
INTRODUCTION

In most engineering problems, we need to find the values of a field variable such as displacement, stress, temperature, pressure, and velocity as a function of spatial coordinate *(x,y,*z). In the case of transient or unsteady state problems, the field variable has to b found as a function of not only the spatial coordinates *(x,y,z)* but also time *(t).* The geometry (domain or solution region) of the problem is often irregular. The first step c the finite element analysis involves the discretization of the irregular domain into small and regular sub domains, known as finite elements. This is equivalent to replacing the domain having an infinite number of degrees of freedom by a system having finite number of degrees of freedom.

A variety of methods can be used to model a domain with finite elements. Different methods of dividing the domain into finite elements involve different amounts of computational time and often lead to different approximations to the solution of the physics problem. The process of discretization is essentially an exercise of engineering judgment. Efficient methods of finite element idealization require some experience and a knowledge of simple guidelines. For large problems involving complex geometries, finite element idealization based on manual procedures requires considerable effort and time on the part of the analyst. Some programs have been developed for the automatic mesh generation for the efficient idealization of complex domains with minimal interface with the analyst.

BASIC ELEMENT SHAPES

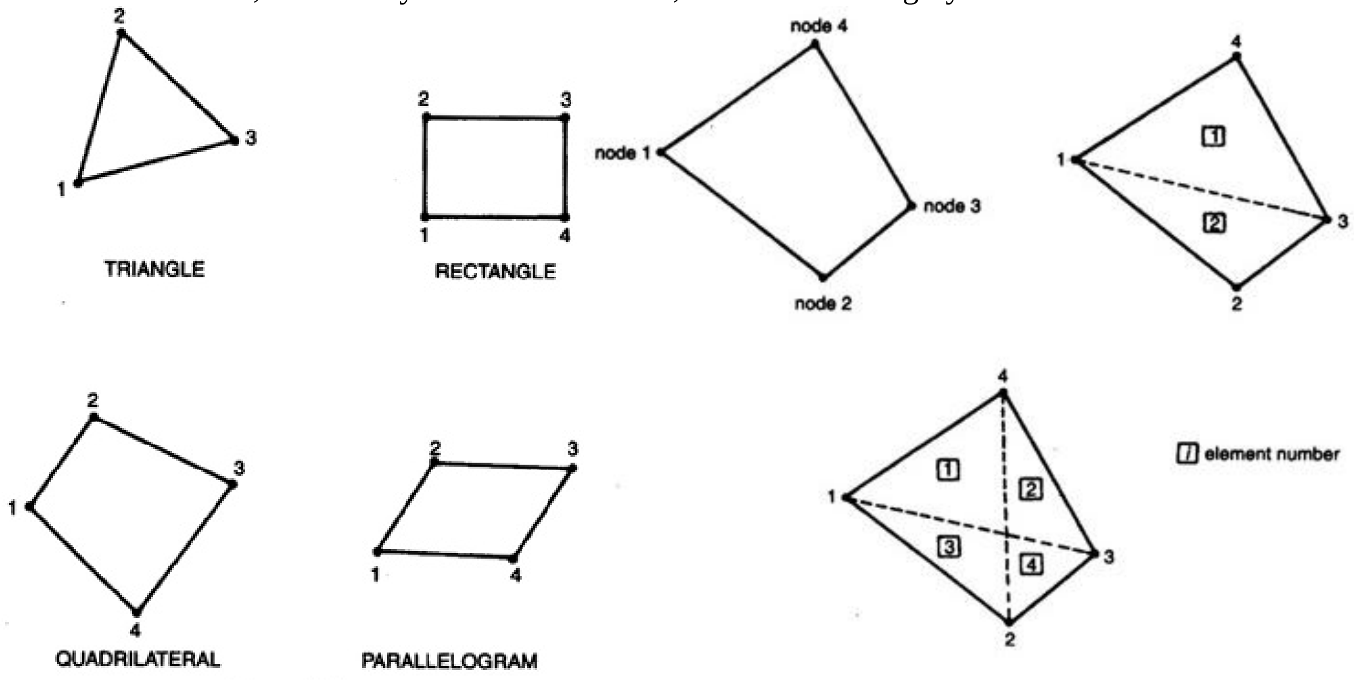
The shapes, sizes, number, and configurations of the elements have to be chosen carefully such that the original body or domain is simulated as closely as possible without increasing the computational effort needed for the solution. Mostly the choice of the type of element is to dictated by the geometry of the body and the number of independent coordinates necessary to describe the system. If the geometry, material properties, and the field variable of the problem can be described in terms of only one spatial coordinate, we can use the one-dimensional or line elements shown in figure. The temperature distribution in a rod (or fin), the pressure distribution in a pipe flow, and the deformation of a bar under axial load, for example, can be determined using these elements. Although these elements have cross-sectional area, they are generally shown schematically as a line element (figure). In some cases, the cross-sectional area of the element may be nonuniform.



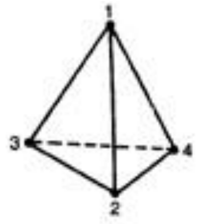
For a simple analysis, one-dimensional elements are assumed to have two nodes, one at each end, with the corresponding value of the field variable chosen as the unknown (degree of freedom). However, for the analysis of beams, the values of the field variable (transverse displacement) and its derivative (slope) are chosen as the unknowns (degrees of freedom) at each node as shown in Figure (c).

When the configuration and other details of the problem can be described in terms of two independent spatial coordinates, we can use the two-dimensional elements shown in Figure. The basic element useful for two-dimensional analysis is the triangular element. Although a quadrilateral (or its special forms, rectangle and parallelogram) element can be obtained by assembling two or four triangular elements, as shown in Figure, in some cases the use of quadrilateral (or rectangle or parallelogram) elements proves to be advantageous. For the bending analysis of plates, multiple degrees of freedom (transverse displacement and its derivatives) are used at each node.

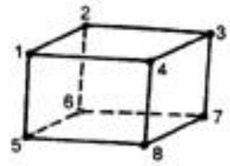
If the geometry, material properties, and other parameters of the body can be described by three independent spatial coordinates, we can idealize the body by using the three-dimensional elements shown in Figure. The basic three-dimensional element, analogous to the triangular element in the case of two-dimensional problems, is the tetrahedron element. In some cases the hexahedron element, which can be obtained by assembling five tetrahedrons as indicated in Figure, can be used advantageously. Some problems, which are actually three-dimensional, can be described by only one or two independent coordinates. Such problems can be idealized by using an axisymmetric or ring type of elements shown in Figure. The problems that possess axial symmetry, such as pistons, storage tanks, valves, rocket nozzles, and reentry vehicle heat shields, fall into this category.



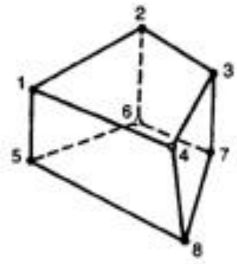
For the discretization of problems involving-curved geometries, finite elements with curved sides are useful. Typical elements having curved boundaries are shown in Figure. The ability to model curved boundaries has been made possible by the addition of midside nodes. Finite elements with straight sides are known as linear elements, whereas those with curved sides are called higher order elements.



TETRAHEDRON



RECTANGULAR PRISM



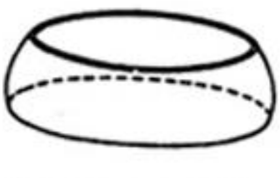
HEXAHEDRON

DISCRETIZATION PROCESS

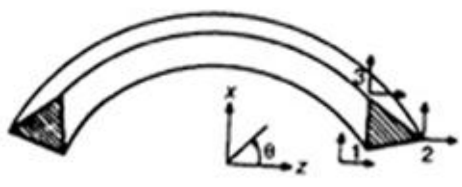
The various considerations to be taken in the discretization process are given in the following sections [2.1].

1) Type of Elements

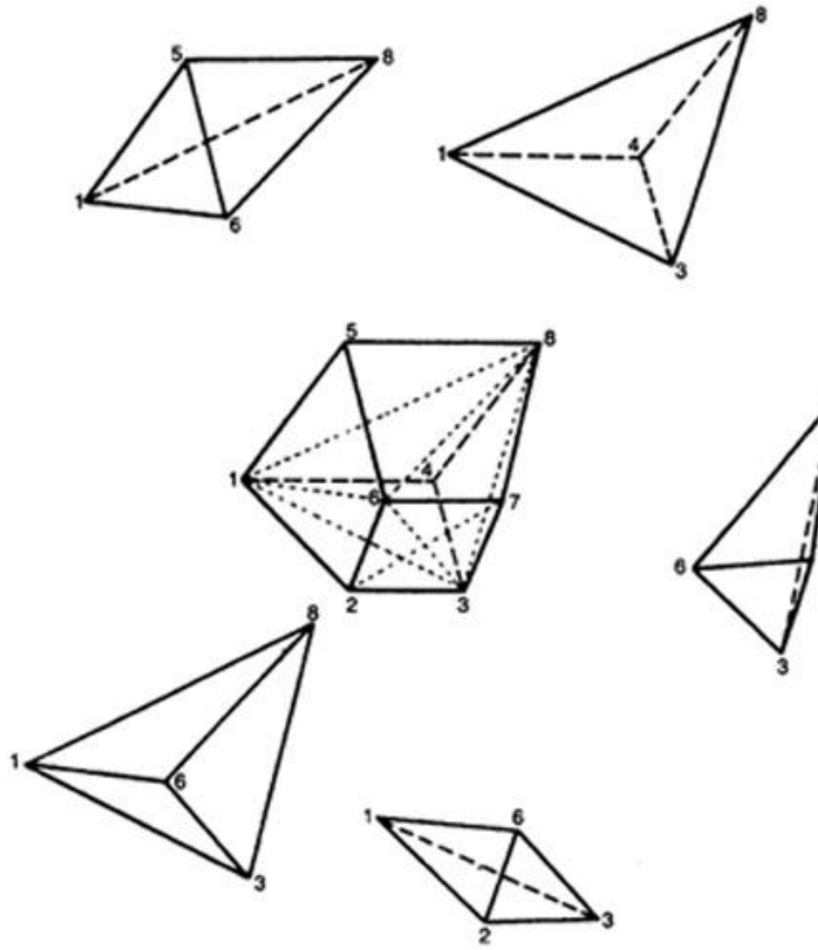
Often, the type of elements to be used will be evident from the physical problem. For example, if the problem involves the analysis of a truss structure under a given set of load conditions (Figure (a)), the type of elements to be used for idealization is obviously the "bar or line elements" as shown in Figure (b). Similarly, in the case of stress analysis of the short beam shown in Figure (a), the finite element idealization can be done using three-dimensional solid elements as shown in Figure (b). However, the type of elements to be used for idealization may not be apparent, and in such cases one has to choose the type of elements judicially. As an example, consider the problem of analysis of the thin-walled shell shown in Figure (a). In this case, the shell can be idealized by several types of elements as shown in Figure (b).

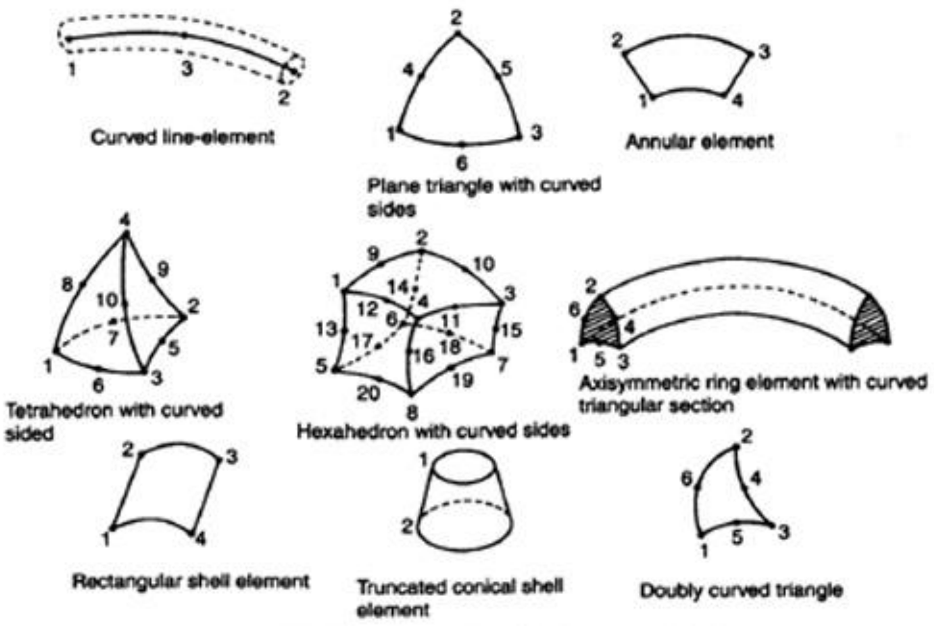


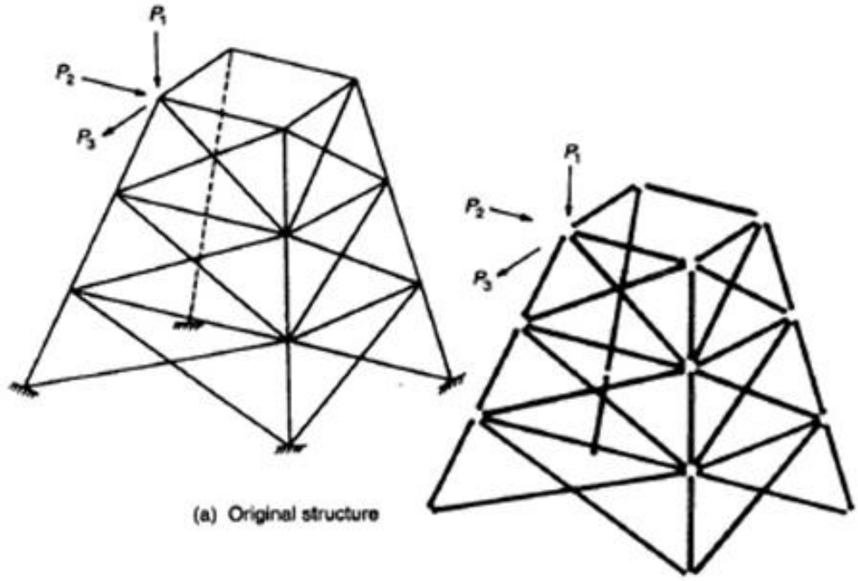
(a) An one dimensional axisymmetric (shell) element



(b) A two dimensional axisymmetric (toroidal) element

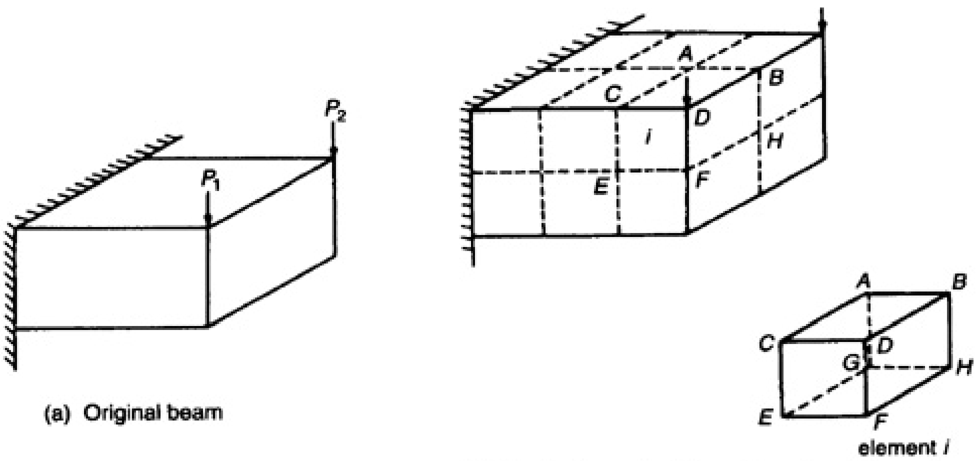



Curved line-element
Plane triangle with curved sides
Annular element
Tetrahedron with curved sided
Hexahedron with curved sides
Axisymmetric ring element with curved triangular section
Rectangular shell element
Truncated conical shell element
Doubly curved triangle



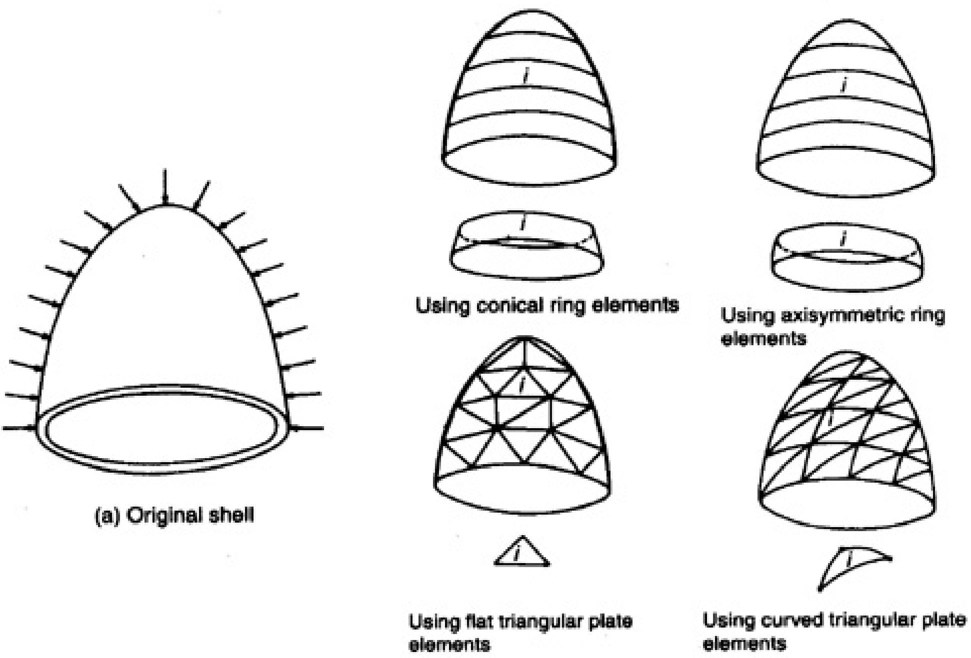
(a) Original structure

(b) Idealization using bar elements



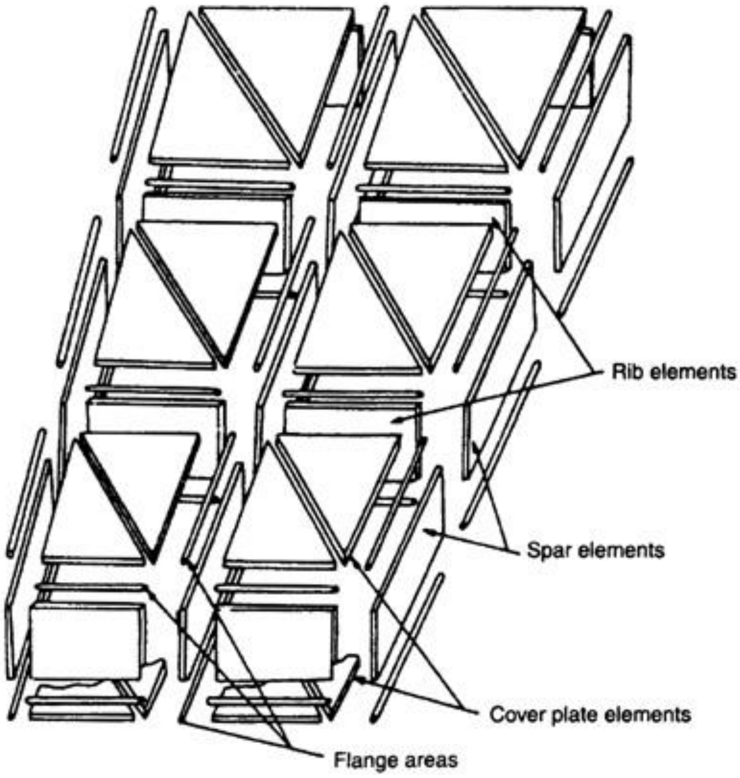
(a) Original beam

(b) Idealization using three-dimensional elements



(a) Original shell

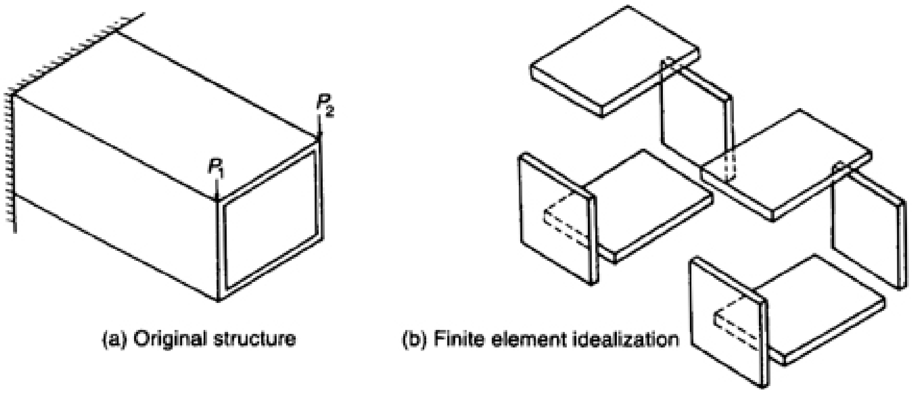
(b) Idealization using different types of elements



Here, the number of degrees of freedom needed, the expected accuracy, the ease with which the necessary equations can be derived, and the degree to which the physical structure can be modeled without approximation will dictate the choice of the element type to be used for idealization. In certain problems, the given body cannot be represented as an assemblage of only one type of elements. In such cases, we may have to use two or more types of elements for idealization. An example of this would be the analysis of an aircraft wing. Since the wing consists of top and bottom covers, stiffening webs, and flanges, three types of elements, namely, triangular plate elements (for covers), rectangular shear panels (for webs), and frame elements (for flanges), have been used in the idealization shown in Figure.

2) Size of Elements

The size of elements influences the convergence of the solution directly and hence it has to be chosen with care. If the size of the elements is small, the final solution is expected to be more accurate. However, we have to remember that the use of elements of smaller size will also mean more computational time. Sometimes, we may have to use elements of different sizes in the same body. For example, in the case of stress analysis of the box beam shown in Figure (a), the size of all the elements can be approximately the same, as shown in Figure (b). However, in the case of stress analysis of a plate with a hole shown in Figure (a), elements of different sizes have to be used, as shown in Figure (b). The size of elements has to be very small near the hole (where stress concentration is expected) compared to far away places. In general, whenever steep gradients of the field variable are expected, we have to use a finer mesh in those regions. Another characteristic related to the size of elements that affects the finite element solution is the aspect ratio of the elements. The aspect ratio describes the shape of the element in the assemblage of elements. For two-dimensional elements, the aspect ratio is taken as the ratio of the largest dimension of the element to the smallest dimension. Elements with an aspect ratio of nearly unity generally yield best results.

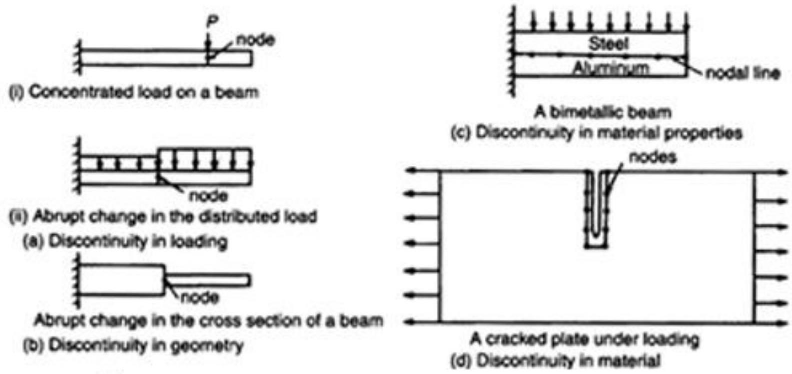


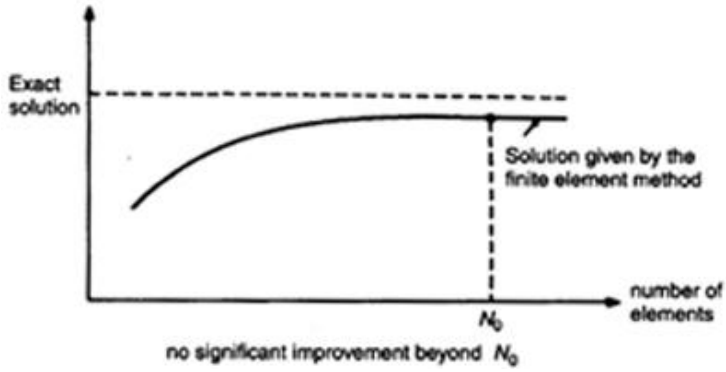
(a) Original structure (b) Finite element idealization

(a) Original structure (b) Idealization using elements of different sizes

3) Location of Nodes

If the body has no abrupt changes in geometry, material properties, and external conditions (e.g., load and temperature), the body can be divided into equal subdivisions and hence the spacing of the nodes can be uniform. On the other hand, if there are any discontinuities in the problem, nodes have to be introduced at these discontinuities, as shown in Figure.





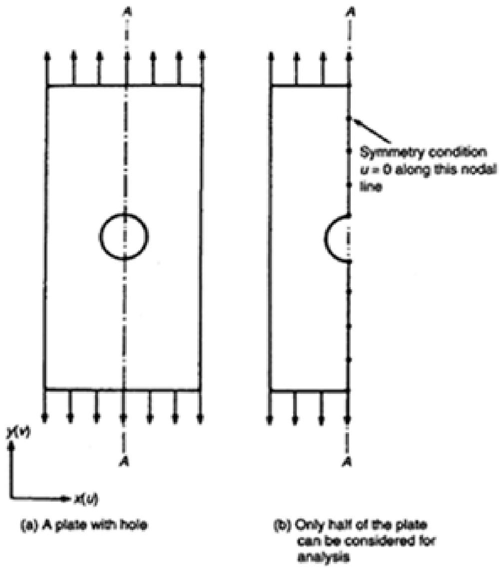
4) Number of Elements

The number of elements to be chosen for idealization is related to the accuracy desired, size of elements, and the number of degrees of freedom involved. Although an increase in the number of elements generally means more accurate results, for any given problem, there will be a certain number of elements beyond which the accuracy cannot be improved by any significant amount. This behavior is shown graphically in Figure.

Moreover, since the use of large number of elements involves a large number of degrees of freedom, we may not be able to store the resulting matrices in the available computer memory.

5) Simplifications Afforded by the Physical Configuration of the Body

If the configuration of the body as well as the external conditions are symmetric, we may consider only half of the body for finite element idealization. The symmetry conditions, however, have to be incorporated in the solution procedure. This is illustrated in Figure, where only half of the plate with hole, having symmetry in both geometry and loading, is considered for analysis. Since there cannot be a horizontal displacement along the line of symmetry *AA*, the condition that $u = 0$ has to be incorporated while finding the solution.



6) Finite Representation of Infinite Bodies

In most of the problems, like in the case of analysis of beams, plates, and shells, the boundaries of the body or continuum are clearly defined. Hence, the entire body ca be considered for the element idealization. However, in some cases, as in the case (analysis of dams, foundations, and semi-infinite bodies, the boundaries axe not clearly defined. In the case of dams (Figure), since the geometry is uniform and the loading does not change in the length direction, a unit slice of the dam can be considered for idealization and analyzed as a plane strain problem. However, in the case of the foundation problem shown in Figure (a), we cannot idealize the complete semi-infinite soil by finite elements. Fortunately, it is not really necessary to idealize the infinite body. Since the effect of loading decreases gradually with increasing distance from the point of loading, we can consider only that much of the continuum in which the loading is expected to have significant effect as shown in Figure (b). Once the significant extent of the infinite body is identified as shown in Figure (b), the boundary conditions for this finite body have to be incorporated in the solution. For example, if the horizontal movement only has to be restrained for sides AB and CD (i.e., u = 0), these sides are supposed to be on rollers as shown in Figure 2.18(b). In this case, the bottom boundary can be either completely fixed (u = v = 0) or constrained only against vertical movement (v = 0). The fixed conditions (u = v = 0 along BC) are often used if the lower boundary is taken at the known location of a bedrock surface.

In Figure the semi-infinite soil has been simulated by considering only a finite portion of the soil. In some applications, the determination of the size of the finite domain may pose a problem. In such cases, one can use infinite elements for modeling [2.3-2.5]. As an example, Figure shows a four-node element that is infinitely long in the x direction. The coordinates of the nodes of this infinite element can be transformed to the natural coordinate system (s, t) as

$$s = 1 - 2\left\{\frac{1}{x} \cdot \frac{(y_3 - y)x_1 + (y - y_1)x_4}{(y_3 - y_1)}\right\}^m ; \quad m \geq 1$$

$$t = 1 - 2\left\{\frac{y_3 - y}{y_3 - y_1}\right\}$$

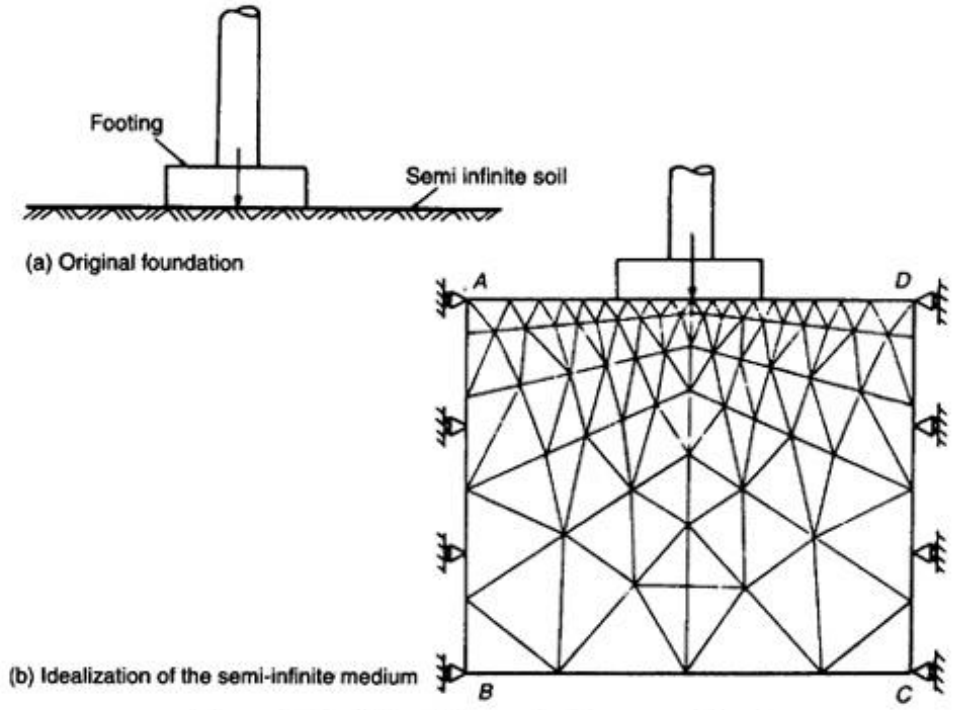
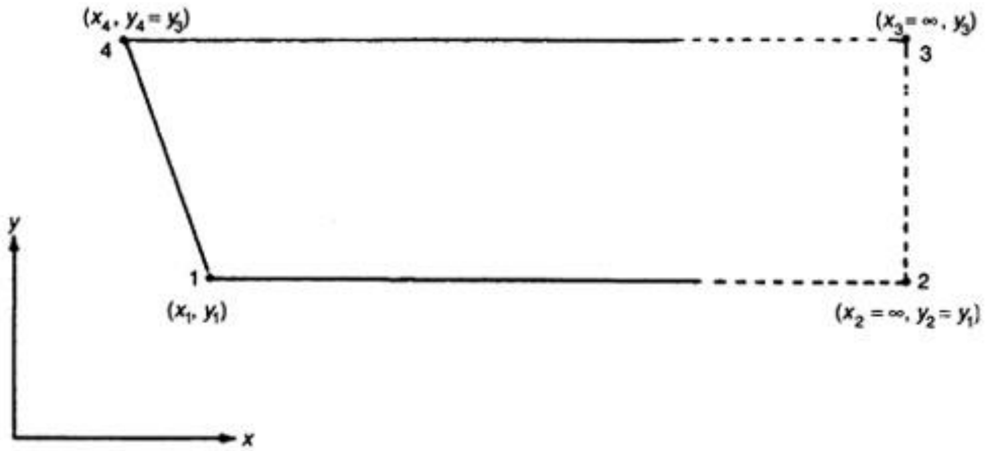


Figure 2.18. A Foundation under Concentrated Load.



AUTOMATIC MESH GENERATION

As indicated in the previous section, the bandwidth of the overall system matrix depends on the manner in which the nodes are numbered. For simple systems or regions, it is easy to label the nodes so as to minimize the bandwidth. But for large systems, the procedure Becomes nearly impossible. Hence, automatic mesh generation algorithms, capable of discretizing any geometry into an efficient finite element mesh without user intervention, have been developed [2.7, 2.8]. Most commercial finite element software has built-in automatic mesh generation codes. An automatic mesh generation program generates the locations of the node points and elements, labels the nodes and elements, and provides the element-node connectivity relationships. The automatic mesh generation schemes are usually tied to solid modeling and computer-aided design schemes. When the user supplies information on the surfaces and volumes of the material domains that make up the object or system, an automatic mesh generator generates the nodes and elements in the object.

The user can also specify the minimum permissible element sizes for different regions of the object.

The most common methods used in the development of automatic mesh generators are the tessellation and octree methods [2.9, 2.10]. In the tessellation method, the user gives a collection of node points and also an arbitrary starting node. The method then creates the first simplex element using the neighboring nodes. Then a subsequent or neighboring element is generated by selecting the node point that gives the least distorted element shape. The procedure is continued until all the elements are generated. The step-by-step procedure involved in this method is illustrated in Figure for a two-dimensional example. Alternately, the user can define the boundary of the object by a series of nodes. Then the tessellation method connects selected boundary nodes to generate simplex elements. The stepwise procedure used in this approach is shown in Figure.

The octree methods belong to a class of mesh generation schemes known as tree structure methods, which are extensively used in solid modeling and computer graphics display methods. In the octree method, the object is first considered enclosed in a three-dimensional cube. If the object does not completely (uniformly) cover the cube, the cube is subdivided into eight equal parts. In the two-dimensional analog of the octree method, known as the quadtree method, the object is first considered enclosed in a square region. If the object does not completely cover the square, the square is subdivided into four equal quadrants. If any one of the resulting quadrants is full (completely occupied by the object) or empty (not occupied by the object), then it is not subdivided further. On the other hand, if any one of the resulting quadrants is partially full (partially occupied by the object), it is subdivided into four quadrants. This procedure of subdividing partially full quadrants is continued until all the resulting regions are either full of empty or until some predetermined level of resolution is achieved. At the final stage, the partially full quadrants are assumed to be either full or empty arbitrarily based on a pre-specified criterion.

Ref.: Finite Element Methods - by S.S. Rao.

QUESTIONS:

EXPERIMENT NO. - 05

AIM: To study about Geometric Modeling and practices on modeling software.